Równania i nierówności wymierne na początek

Zadanie Rozwiąż równanie

a) $\frac{2x}{x+3} = 5$ $D_r: x+3 \neq 0$, $x \neq -3$

Można też napisać tak: $D_r = \mathbb{R} \setminus \{-3\}$

$\frac{2x}{x+3} = \frac{5}{1}$

$2x \cdot 1 = 5 \cdot (x+3)$

$2x = 5x + 15$

$2x - 5x = 15$

$-3x = 15 \quad | : (-3)$

$x = -5 \in D_r$

Odp.: $x \in \{-5\}$

b) $\frac{3}{x-2} = x+2$ $D = \mathbb{R} \setminus \{2\}$

$\frac{3}{x-2} = \frac{x+2}{1}$

$(x-2) \cdot (x+2) = 3 \cdot 1$

$x^2 - 4 = 3$

$x^2 = 7$

$x = \sqrt{7} \vee x = -\sqrt{7}$

$x = \sqrt{7} \in D_r$, $x = -\sqrt{7} \in D_r$

Odp.: $x \in \{\sqrt{7}, -\sqrt{7}\}$

c) $\frac{1-x}{x+3} = 2x+3$ $D_r = \mathbb{R} \setminus \{-3\}$

$\frac{1-x}{x+3} = \frac{2x+3}{1}$

$(x+3) \cdot (2x+3) = (1-x) \cdot 1$

$2x^2 + 3x + 6x + 9 = 1 - x$

$2x^2 + 9x + 9 - 1 + x = 0$

$2x^2 + 10x + 8 = 0 \quad | : 2$

$x^2 + 5x + 4 = 0$

$\Delta = 25 - 16 = 9$, $\sqrt{\Delta} = 3$

$x_1 = \frac{-5-3}{2} = -\frac{8}{2} = -4 \quad \vee \quad x_2 = \frac{-5+3}{2} = -\frac{2}{2} = -1$

$x_1 = -4 \in D_r$, $x_2 = -1 \in D_r$

Odp.: $x \in \{-4, -1\}$

d) $\frac{x-1}{x+2} - \frac{x}{x-1} = 0$ $D_r = \mathbb{R} \setminus \{-2, 1\}$

$\frac{x-1}{x+2} = \frac{x}{x-1}$

$(x-1) \cdot (x-1) = x \cdot (x+2)$

$(x-1)^2 = x \cdot (x+2)$

$x^2 - 2x + 1 = x^2 + 2x$

$\cancel{x^2} - 2x + 1 - \cancel{x^2} - 2x = 0$

$-4x = -1 \quad | : (-4)$

$x = \frac{1}{4} \in D_r$

Odp.: $x \in \{\frac{1}{4}\}$

e) $\frac{x-2}{x-3} = 0$ $D_r = \mathbb{R} \setminus \{3\}$

$x - 2 = 0$

$x = 2 \in D_r$

Odp.: $x \in \{2\}$

f) $\frac{x^2-4}{x-2} = 0$ $D_r = \mathbb{R} \setminus \{2\}$

$x^2 - 4 = 0$

$x^2 = 4$

$x = 2 \notin D_r \quad \vee \quad x = -2 \in D_r$

Odp.: $x \in \{-2\}$

g) $\frac{1-x}{x} - \frac{x+2}{x+3} = -2$ $D_r = \mathbb{R} \setminus \{0, -3\}$

$\frac{(1-x) \cdot (x+3)}{x \cdot (x+3)} - \frac{(x+2) \cdot x}{x \cdot (x+3)} = -2$

$\frac{(1-x) \cdot (x+3) - (x+2) \cdot x}{x \cdot (x+3)} = -2$

$\frac{x + 3 - x^2 - 3x - [x^2 + 2x]}{x(x+3)} = -2$

$\frac{x + 3 - x^2 - 3x - x^2 - 2x}{x(x+3)} = -2$

$\frac{-2x^2 - 4x + 3}{x \cdot (x+3)} = -\frac{2}{1}$

$-2x^2 - 4x + 3 = -2 \cdot x \cdot (x+3)$

$-2x^2 - 4x + 3 = -2x^2 - 6x$

$-\cancel{2x^2} - 4x + 3 + \cancel{2x^2} + 6x = 0$

$2x = -3 \quad | : 2$

$x = -\frac{3}{2} \in D_r$

Odp.: $x \in \{-\frac{3}{2}\}$

h) $\frac{2x-1}{x-1} - 2x - 1 = 0$ $D_r = \mathbb{R} \setminus \{1\}$

$\frac{2x-1}{x-1} = 2x+1$

$\frac{2x-1}{x-1} = \frac{2x+1}{1}$

$(2x-1) \cdot 1 = (2x+1) \cdot (x-1)$

$2x - 1 = 2x^2 - 2x + x - 1$

$2x^2 - 2x + x - \cancel{1} - 2x + \cancel{1} = 0$

$2x^2 - 3x = 0$

$x \cdot (2x - 3) = 0$

$x = 0 \quad \vee \quad 2x - 3 = 0$

$2x = 3 \quad | : 2$

$x = \frac{3}{2} \in D_r$

$x = 0 \in D_r$ ✓

Odp.: $x \in \{0, \frac{3}{2}\}$

i) $\frac{x-3}{3x+2} = x+1$ $D_r = \mathbb{R} \setminus \{-\frac{2}{3}\}$

$3x + 2 \neq 0$

$3x \neq -2 \quad | : 3$

$x \neq -\frac{2}{3}$

$\frac{x-3}{3x+2} = \frac{x+1}{1}$

$(3x+2) \cdot (x+1) = (x-3) \cdot 1$

$3x^2 + 3x + 2x + 2 = x - 3$

$3x^2 + 5x + 2 - x + 3 = 0$

$3x^2 + 4x + 5 = 0$

$\Delta = 16 - 4 \cdot 3 \cdot 5 = (16 - 60) < 0$

$\Delta < 0$

Odp.: Brak rozwiązań.

j) $\frac{1}{x} = 2$ $D_r = \mathbb{R} \setminus \{0\}$

$\frac{1}{x} = \frac{2}{1}$

$2x = 1$

$x = \frac{1}{2} \in D_r$

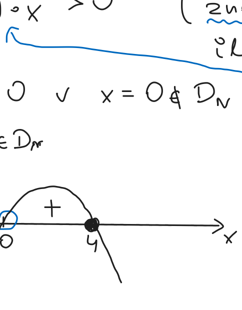
Odp.: $x \in \{\frac{1}{2}\}$

Zadanie Rozwiąż nierówność

a) $\frac{x-1}{x-2} \geq 0 \quad / \cdot (x-2)^2$ $D_N = \mathbb{R} \setminus \{2\}$

$(x-1) \cdot (x-2) \geq 0$ i $x \in D_N$

$x = 1 \vee x = 2$



Odp.: $x \in (-\infty, 1\rangle \cup (2, +\infty)$ – po uwzględnieniu dziedziny.

b) $\frac{4}{x} > 1$ $D_N = \mathbb{R} \setminus \{0\}$

$\frac{4}{x} - 1 > 0$

$\frac{4}{x} - \frac{x}{x} > 0$

$\frac{4-x}{x} > 0$

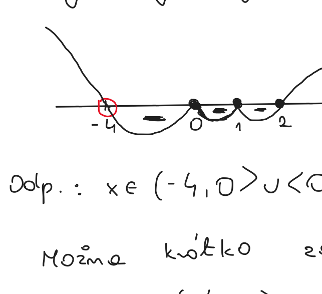
$(4-x) \cdot x > 0$ (znak ilorazu jest zgodny ze znakiem iloczynu)

$4x - x^2 > 0$

$-x^2 + 4x > 0$

$4 - x = 0 \quad \vee \quad x = 0 \notin D_N$

$x = 4 \in D_r$



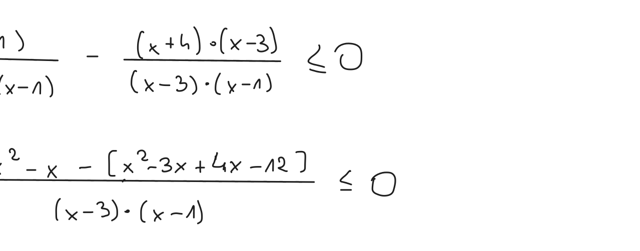
Odp.: $x \in (0, 4\rangle$ – po uwzględnieniu dziedziny.

c) $\frac{(x-1)^4 \cdot x^2 \cdot (2-x)}{(x-7)^3 \cdot (x+4) \cdot (7-x)} \leq 0$ $D_N: \begin{cases} x - 7 \neq 0 \\ x + 4 \neq 0 \\ 7 - x \neq 0 \end{cases} \Leftrightarrow \begin{cases} x \neq 7 \\ x \neq -4 \\ x \neq 7 \end{cases}$

$D_N = \mathbb{R} \setminus \{-4, 7\}$

$(x-1)^4 \cdot x^2 \cdot (2-x)^1 \cdot (x-7)^3 \cdot (x+4)^1 \cdot (7-x)^1 \leq 0$ $(+x^{12})$

$x = 1 \vee x = 0 \vee x = 2 \vee x = 7 \vee x = -4 \vee x = 7$



Odp.: $x \in (-4, 0\rangle \cup \langle 0, 1\rangle \cup \langle 1, 2\rangle$

Można krótko zapisać tak:

$x \in (-4, 2\rangle$

d) $\frac{x}{x-3} \leq \frac{x+4}{x-1}$ $D_N = \mathbb{R} \setminus \{3, 1\}$

$\frac{x}{x-3} - \frac{x+4}{x-1} \leq 0$

$\frac{x(x-1)}{(x-3) \cdot (x-1)} - \frac{(x+4) \cdot (x-3)}{(x-3) \cdot (x-1)} \leq 0$

$\frac{x^2 - x - [x^2 - 3x + 4x - 12]}{(x-3) \cdot (x-1)} \leq 0$

$\frac{\cancel{x^2} - x - \cancel{x^2} + 3x - 4x + 12}{(x-3) \cdot (x-1)} \leq 0$

$\frac{-2x + 12}{(x-3)(x-1)} \leq 0$

$\frac{-2(x-6)}{(x-3)(x-1)} \leq 0$

$-2(x-6) \cdot (x-3) \cdot (x-1) \leq 0$ $(-x^3)$

$x = 6 \vee x = 3 \vee x = 1$

Odp.: $x \in (1, 3) \cup \langle 6, +\infty)$

e) $-\frac{1}{x} < \frac{4}{x+3}$ $D_N = \mathbb{R} \setminus \{0, -3\}$

$-\frac{1}{x} - \frac{4}{x+3} < 0$

$\frac{-1(x+3)}{x(x+3)} - \frac{4 \cdot x}{x(x+3)} < 0$

$\frac{-x - 3 - 4x}{x(x+3)} < 0$

$\frac{-5x - 3}{x(x+3)} < 0$

$(-5x - 3) \cdot x \cdot (x+3) < 0$ $(-x^3)$

$-5x - 3 = 0 \quad \vee \quad x = 0 \quad \vee \quad x = -3$

$-5x = 3 \quad | : (-5)$

$x = -\frac{3}{5}$

Odp.: $x \in (-3, -\frac{3}{5}) \cup (0, +\infty)$