Funkcja wykładnicza cz.1

Potęga o wykładniku wymiernym

Dla $a, b > 0$ i $x, y \in \mathbb{W}$

1) $a^x \cdot a^y = a^{x+y}$

2) $\frac{a^x}{a^y} = a^{x-y}$

3) $(a^x)^y = a^{x \cdot y}$

4) $a^x \cdot b^x = (a \cdot b)^x$

5) $\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$

Zadanie Przedstaw liczbę w postaci 2^m:

1) $2^1 \cdot 2^4 \cdot 2^5 = 2^{1+4+5} = 2^{10}$

2) $2^{-4} : 2^{-6} = 2^{-4-(-6)} = 2^{-4+6} = 2^2$

3) $(2^2)^3 \cdot 2 = 2^6 \cdot 2^1 = 2^7$

4) $2^1 \cdot 2^{15} : 2^3 = 2^{16} : 2^3 = 2^{16-3} = 2^{13}$

5) $2^{-6} : (2^6 : 2^{-4}) = 2^{-6} : \left(2^{6-(-4)}\right) =$

$= 2^{-6} : 2^{6+4} = 2^{-6} : 2^{10} = 2^{-6-10} = 2^{-16}$

6) $4^3 \cdot 2^4 : 8^{-4} = (2^2)^3 \cdot 2^4 : (2^3)^{-4} =$

$= 2^6 \cdot 2^4 : 2^{-12} = 2^{10} : 2^{-12} = 2^{10-(-12)} = 2^{22}$

7) $(16^{-2} : 2^{-3}) \cdot 4 = \left((2^4)^{-2} : 2^{-3}\right) \cdot 2^2 =$

$= \left(2^{-8} : 2^{-3}\right) \cdot 2^2 = 2^{-8-(-3)} \cdot 2^2 = 2^{-8+3} \cdot 2^2 =$

$= 2^{-5} \cdot 2^2 = 2^{-3}$

8) $(8^{-6} : 4^{-6})^{-1} = \left((2^3)^{-6} : (2^2)^{-6}\right)^{-1} =$

$= (2^{-18} : 2^{-12})^{-1} = \left(2^{-18-(-12)}\right)^{-1} = \left(2^{-18+12}\right)^{-1} =$

$= (2^{-6})^{-1} = 2^6$

Zadanie Oblicz

1) $49^{\frac{1}{2}} = \sqrt[2]{49} = \sqrt{49} = 7$

2) $\left(\frac{4}{25}\right)^{\frac{1}{2}} = \sqrt{\frac{4}{25}} = \frac{2}{5}$

3) $64^{\frac{1}{3}} = \sqrt[3]{64} = 4$

4) $\left(\frac{8}{27}\right)^{\frac{1}{3}} = \sqrt[3]{\frac{8}{27}} = \frac{2}{3}$

5) $\left(\frac{25}{16}\right)^{\frac{3}{2}} = \sqrt[2]{\left(\frac{25}{16}\right)^3} = \sqrt{\frac{25}{16} \cdot \frac{25}{16} \cdot \frac{25}{16}} =$

$= \sqrt{\frac{25}{16}} \cdot \sqrt{\frac{25}{16}} \cdot \sqrt{\frac{25}{16}} = \frac{5}{4} \cdot \frac{5}{4} \cdot \frac{5}{4} = \frac{125}{64}$

6) $\left(\frac{1}{2}\right)^{-1} = \frac{1}{\left(\frac{1}{2}\right)^1} = 1 : \frac{1}{2} = 1 \cdot 2 = 2$

7) $\left(\frac{3}{2}\right)^{-2} = \frac{1}{\left(\frac{3}{2}\right)^2} = \frac{1}{\frac{9}{4}} = 1 : \frac{9}{4} = \frac{4}{9}$

8) $3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

9) $\left(2\frac{1}{4}\right)^{-\frac{3}{2}} = \frac{1}{\left(2\frac{1}{4}\right)^{\frac{3}{2}}} = \frac{1}{\left(\frac{9}{4}\right)^{\frac{3}{2}}} = \frac{1}{\sqrt[2]{\left(\frac{9}{4}\right)^3}} =$

$= \frac{1}{\sqrt{\frac{9}{4} \cdot \frac{9}{4} \cdot \frac{9}{4}}} = \frac{1}{\frac{3}{2} \cdot \frac{3}{2} \cdot \frac{3}{2}} = \frac{1}{\frac{27}{8}} = \frac{8}{27}$

Własności:

$$a^{\frac{1}{n}} = \sqrt[n]{a^1}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

Zadanie Przedstaw liczbę w postaci a^m, $m \in W$.

1) $7^{\frac{1}{3}} \cdot 7^{\frac{1}{2}} = 7^{\frac{1}{3} + \frac{1}{2}} = 7^{\frac{2}{6} + \frac{3}{6}} = 7^{\frac{5}{6}}$

2) $3^{\frac{5}{2}} \cdot 3^{\frac{3}{4}} = 3^{\frac{5}{2} + \frac{3}{4}} = 3^{\frac{10}{4} + \frac{3}{4}} = 3^{\frac{13}{4}}$

3) $\left(\frac{2}{3}\right)^{\frac{1}{2}} \cdot \left(\frac{2}{3}\right)^{-\frac{1}{3}} = \left(\frac{2}{3}\right)^{\frac{1}{2} + \left(-\frac{1}{3}\right)} = \left(\frac{2}{3}\right)^{\frac{3}{6} - \frac{2}{6}} = \left(\frac{2}{3}\right)^{\frac{1}{6}}$

4) $3^{\frac{2}{3}} \cdot 3^{-\frac{4}{3}} \cdot 9^2 = 3^{\frac{2}{3} + \left(-\frac{4}{3}\right)} \cdot \left(3^2\right)^2 =$

$= 3^{\frac{2}{3} - \frac{4}{3}} \cdot 3^4 = 3^{-\frac{2}{3}} \cdot 3^4 = 3^{-\frac{2}{3} + 4} = 3^{3\frac{1}{3}} = 3^{\frac{10}{3}}$

5) $25^{-0{,}5} \cdot \left(5^{\frac{1}{3}}\right)^{-6} = \left(5^{\overset{1}{\cancel{2}}}\right)^{-\frac{1}{\underset{1}{\cancel{2}}}} \cdot 5^{\frac{1}{\underset{1}{\cancel{3}}} \cdot (-\overset{2}{\cancel{6}})} =$

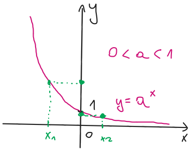
$= 5^{-1} \cdot 5^{-2} = 5^{-1+(-2)} = 5^{-3}$

Funkcja wykładnicza

Jest to funkcja postaci $f(x)=a^{x}$, gdzie $a>0, a\neq 1$ i $x\in\mathbb{R}$.

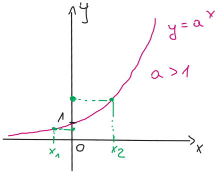
Rozpatrujemy dwa przypadki:

1) $a\in(0,1)$, $x\in\mathbb{R}$



f. malejąca

2) $a>1$, $x\in\mathbb{R}$



f. rosnąca

f. różnowartościowa

Zadanie Rozwiąż równanie

1) $2^{x+1}=2^{4x-1}$ $\qquad D_r=\mathbb{R}$

Z różnowartościowości funkcji wykładniczej

$x+1=4x-1$

$x-4x=-1-1$

$-3x=-2 \quad |:(-3)$

Odp.: $x=\frac{2}{3}\in D_r$

2) $2^{x^2}=4^{2x-1}$ $\qquad D_r=\mathbb{R}$

$2^{x^2}=(2^2)^{2x-1}$

$2^{x^2}=2^{4x-2}$

$x^2=4x-2$

$x^2-4x+2=0$

$\Delta=16-8=8$, $\sqrt{\Delta}=\sqrt{4\cdot 2}=2\sqrt{2}$

$x_1=\frac{4-2\sqrt{2}}{2} \quad \vee \quad x_2=\frac{4+2\sqrt{2}}{2}$

Odp.: $x_1 = 2-\sqrt{2} \quad \vee \quad x_2 = 2+\sqrt{2}$

3) $\left(\frac{1}{3}\right)^{2-x} = 9^{x+3}$

$(3^{-1})^{2-x} = (3^2)^{x+3}$

$3^{-2+x} = 3^{2x+6}$

$-2+x = 2x+6$

$x-2x = 6+2$

$-x = 8$

Odp.: $x = -8$

4) $4^{2x} \cdot 2^{2x+3} = \left(\frac{1}{2}\right)^{2x-1}$

$(2^2)^{2x} \cdot 2^{2x+3} = (2^{-1})^{2x-1}$

$2^{4x} \cdot 2^{2x+3} = 2^{-2x+1}$

$2^{4x+2x+3} = 2^{-2x+1}$

$2^{6x+3} = 2^{-2x+1}$

$6x+3 = -2x+1$

$6x+2x = 1-3$

$8x = -2 \quad / : 8$

$x = -\frac{2}{8}$

Odp.: $x = -\frac{1}{4}$

5) $2 \cdot 9^{1-x} + 4 \cdot 3^{2-2x} - 3 \cdot \left(\frac{1}{3}\right)^{2x-2} = 1$

$D_r = \mathbb{R}$

$2 \cdot (3^2)^{1-x} + 4 \cdot 3^{2-2x} - 3 \cdot \underbrace{(3^{-1})^{2x-2}} = 1$

$2 \cdot 3^{2-2x} + 4 \cdot 3^{2-2x} - 3^{2-2x} \cdot 3 = 1$

$$3^{2-2x}\cdot(2+4-3)=1$$

$$3^{2-2x}\cdot 3^{1}=1$$

$$3^{2-2x+1}=3^{0}$$

$$3^{3-2x}=3^{0}$$

$$3-2x=0$$

$$-2x=-3 \quad /:(-2)$$

odp.: $x=\frac{3}{2}$

6) $3^{x+2}-2\cdot 3^{x}=63$ $\quad D_r=\mathbb{R}$

$$3^{x}\cdot 3^{2}-2\cdot 3^{x}=63$$

$$9\cdot 3^{x}-2\cdot 3^{x}=63$$

$$3^{x}\cdot(9-2)=63$$

$$3^{x}\cdot 7=63 \quad /:7$$

$$3^{x}=9$$

$$3^{x}=3^{2}$$

odp.: $x=2$

7) $4^{x}-3\cdot 2^{x}+2=0$

$$(2^{2})^{x}-3\cdot 2^{x}+2=0$$

$$(2^{x})^{2}-3\cdot 2^{x}+2=0$$

Podstawiamy $t=2^{x}$, $t>0$

$$t^{2}-3t+2=0$$

$$\Delta=9-8=1, \quad \sqrt{\Delta}=1$$

$$t_1=\frac{3-1}{2}=\frac{2}{2}=1 \quad \vee \quad t_2=\frac{3+1}{2}=\frac{4}{2}=2$$

Wracamy do podstawienia

$t_1 = 1 \quad \vee \quad t_2 = 2$

$2^{x_1} = 1 \qquad 2^{x_2} = 2^1$

$2^{x_1} = 2^0 \quad \vee \quad x_2 = 1$

$x_1 = 0$

Odp.: $x_1 = 0, \ x_2 = 1$

8) $25^x - 4 \cdot 5^x - 5 = 0 \qquad D_r = \mathbb{R}$

$(5^2)^x - 4 \cdot 5^x - 5 = 0$

$(5^x)^2 - 4 \cdot 5^x - 5 = 0$

$t = 5^x, \ t > 0$

$t^2 - 4t - 5 = 0$

$\Delta = 16 + 20 = 36, \ \sqrt{\Delta} = 6$

$t_1 = \frac{4-6}{2} = \frac{-2}{2} = -1$ - odpada

$t_2 = \frac{4+6}{2} = 5$

Wracamy do podstawienia

$t = 5$

$5^x = 5^1$

Odp.: $x = 1$

9) $3^{2x+1} + 2 \cdot 3^x - 1 = 0 \qquad D_r = \mathbb{R}$

$3^{2x} \cdot 3^1 + 2 \cdot 3^x - 1 = 0$

$(3^x)^2 \cdot 3 + 2 \cdot 3^x - 1 = 0$

$t = 3^x, \ t > 0$

$3t^2 + 2t - 1 = 0$

$\Delta = 4 + 12 = 16, \ \sqrt{\Delta} = 4$

$t_1 = \frac{-2-4}{6} = \frac{-6}{6} = -1$ - odpada

$t_2 = \frac{-2+4}{6} = \frac{2}{6} = \frac{1}{3}$

Wracamy do podstawienia:

$t = \frac{1}{3}$

$3^x = 3^{-1}$

odp.: $x = -1$

10) $2^x + \frac{1}{2^x} = 2$ $D_r: 2^x \neq 0 \Leftrightarrow x \in \mathbb{R}$

$t = 2^x, t > 0$

$t + \frac{1}{t} = 2 \quad / \cdot t$

$t^2 + 1 = 2t$

$t^2 - 2t + 1 = 0$

$\Delta = 4 - 4 = 0$

$t = -\frac{b}{2a}$

$t = \frac{2}{2} = 1$

Wracamy do podstawienie

$t = 2^x$

$t = 1$

$2^x = 1$

$2^x = 2^0$

odp.: $\underline{x = 0}$