1) $\log_3 2x = \frac{1}{2}$ D: $2x>0$, $x>0$, $D=(0,+\infty)$

$2x = 3^{\frac{1}{2}}$

$2x = \sqrt{3}$

$x = \frac{\sqrt{3}}{2} \in D$

$x \in \{\frac{\sqrt{3}}{2}\}$ zbiór rozwiązań równania

2) $\log_4(x+3) - \log_4(x-1) = 2 - \log_4 8$ D: $x+3>0 \wedge x-1>0$, $x>-3 \wedge x>1$, $x \in (1,\infty)$, $D=(1,\infty)$

$\log_4\left(\frac{x+3}{x-1}\right) = \log_4\left(\frac{16}{8}\right)$

$\frac{x+3}{x-1} = 2$

$x+3 = x-2$

$-x = -5$

$x = 5 \in D$

3) $\log_3(\log_2 x) = 0$ D: $x>0 \wedge \log_2 x > 0$, $x>0 \wedge \log_2 x > \log_2 1$, $x>0 \wedge x>1$, $D=(1,+\infty)$

$\log_2 x = 3^0$

$\log_2 x = 1$

$x = 2^1$

$x = 2 \in D$

$y = \log_2 x$ ↗ bo podstawa $2>1$

$\log_2(a) > \log_2(b)$

$a > b$

$2^0 = 1$ $\log_2 1 = 0$

4) $\log_3^2 x - \log_3 x^3 + 2 = 0$ D: $x>0 \wedge x^3>0$, $x>0 \wedge x>0$, $D=(0,+\infty)$

$(\log_3 x)^2 - \log_3(x^3) + 2 = 0$

$(\log_3 x)^2 - 3\log_3 x + 2 = 0$

$t = \log_3 x$

$t^2 - 3t + 2 = 0$

$\Delta = 9 - 8 = 1$

$t = \frac{3-1}{2} = 1 \vee t = \frac{3+1}{2} = 2$

$\log_3 x = 1 \vee \log_3 x = 2$

$x = 3 \in D \vee x = 9 \in D$

$x \in \{3;9\}$

t_1 t_2 $t = \ldots \vee t = \ldots$

5) $\log_2(x^2-x) > 1$ $1 = 2^1 = 2$

$\log_2(x^2-x) > \log_2 2$

$y = \log_2 x$ ↗

$x^2 - x > 2$

$x^2 - x - 2 > 0$

$\Delta = 1+8 = 9$ $\sqrt{\Delta} = 3$

$x = \frac{1-3}{2} = -1 \vee x = \frac{1+3}{2} = 2$

$x \in (-\infty,-1) \cup (2,+\infty)$

D: $x^2 - x > 0$

$x(x-1) > 0$

$x=0$ $x=1$

$x \in (-\infty,0) \cup (1,+\infty)$

$D = (-\infty,0) \cup (1,+\infty)$

6) $\log_4 x - \log_2 5 \leq 2$ D: $x>0$, $D=(0,+\infty)$

$\log_4 x \leq 2 + \log_2 5$

$\log_4 x \leq \log_2 4 + \log_2 5$

$\log_4 x \leq \log_2 20$

$\log_{2^2} x \leq \log_2 20$

PD

7) $f(x) = \log_4(4x-x^2) + \sqrt{|x+1|-3}$

D: $4x - x^2 > 0 \wedge |x+1| - 3 \geq 0$

$x(4-x) > 0$

$|x+1| \geq 3$

$x \in (0,4)$

$x+1 \geq 3 \vee x+1 \leq -3$

$x \geq 2 \vee x \leq -4$

$x \in (-\infty,-4] \cup [2,\infty)$

$x \in [2,4)$

$D_f = [2,4)$

$\log_2(x^2-x) > \log_2 2$

$x^2 - x > 2$

$x^2 - x$ - argument

$\log_2(x^2-x)$

$y = \log_2 x$ $\log_2 2 = 1$

$f \nearrow$ rosn. $x_1 < x_2 \Leftrightarrow f(x_1) < f(x_2)$

$f \searrow$ $x_1 < x_2 \Leftrightarrow f(x_1) > f(x_2)$

$y = \ln x$

$\ln x = \log_e x$

$e \simeq 2{,}718\ldots$

$C \in \mathbb{R}$ $\ln C \in \mathbb{R}$ $C > 0$

$y = \ln x$

$\ln C$ C

$y' = \frac{1}{x}$

$\ln|y| = \ln|x| + C$ dowolna liczba rzeczywista $C \in \mathbb{R}$

$\ln|y| = \ln|x| + \ln|C|$ $C \in \mathbb{R} \setminus \{0\}$

$\ln|y| = \ln(|x| \cdot |C|)$

$|y| = |x| \cdot |C|$