Zbiory na płaszczyźnie

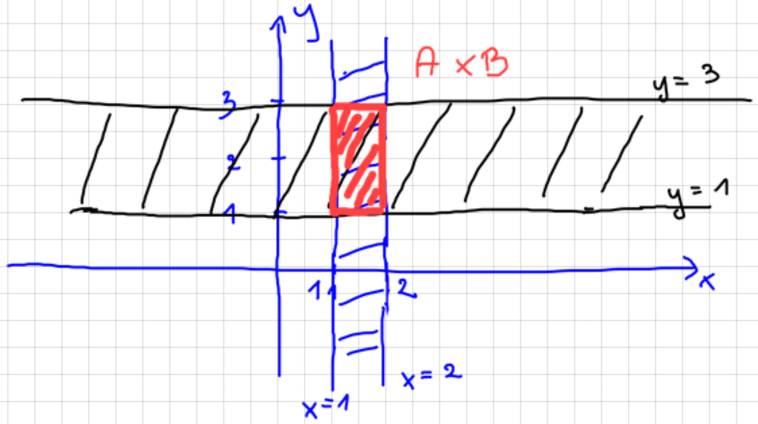
Iloczyn kartezjański zbiorów A i B to zbiór:

$$A \times B = \{\underbrace{(x,y)}_{\text{uporządkowana para punktów}} : \quad x \in A \ \wedge \ y \in B\}$$

$$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(x,y) : \quad x \in \mathbb{R} \ \wedge \ y \in \mathbb{R}\}$$

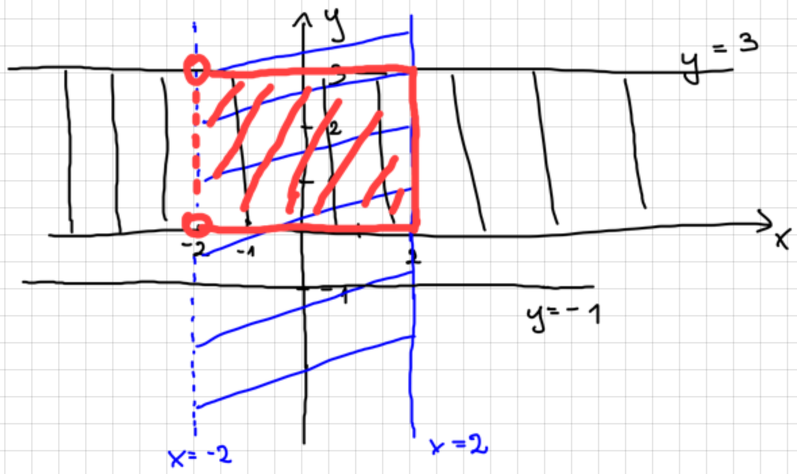
Przykład Na płaszczyźnie zaznaczymy zbiór $A \times B$, gdy

a) $A = \langle 1, 2 \rangle$, $B = \langle 1, 3 \rangle$



$$A \times B = \langle 1,2 \rangle \times \langle 1, 3 \rangle = \{(x,y) : \quad x \in \langle 1,2 \rangle \ \wedge \ y \in \langle 1, 3 \rangle\}$$

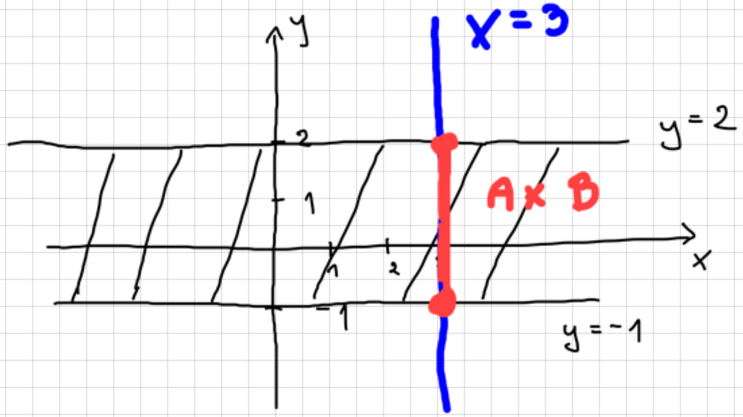
b) $A = (-2, 2\rangle$, $B = \langle -1, 3\rangle$



$$A \times B = (-2,2\rangle \times \langle -1,3\rangle = \{(x,y) \in \mathbb{R}^2 \ / \ x \in (-2,2\rangle \wedge y \in \langle -1,3\rangle\}$$

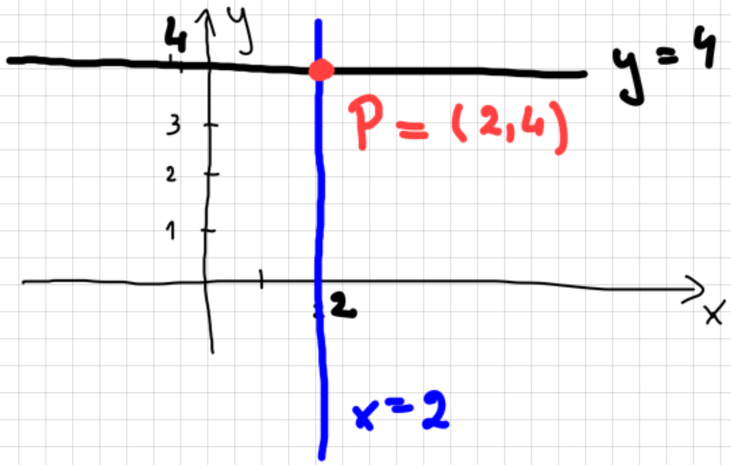
c) $A = \{3\}$ $B = \langle -1, 2\rangle$

$$A \times B = \{3\} \times \langle -1,2\rangle = \{(x,y) \in \mathbb{R}^2 \ / \ x = 3 \wedge y \in \langle -1,2\rangle\}$$



d) $A = \{2\}$ $B = \{4\}$

$A \times B = \{2\} \times \{4\} = \{(x,y) \in \mathbb{R}^2 : x = 2 \wedge y = 4\}$



Przypomnienie

- Równanie okręgu o środku w $S = (x_0, y_0)$ i promieniu $r > 0$:

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

- Nierówność opisująca koło o środku w $S = (x_0, y_0)$ i $r > 0$:

a) koło z brzegiem: $(x - x_0)^2 + (y - y_0)^2 \leq r^2$

b) koło bez brzegu: $(x - x_0)^2 + (y - y_0)^2 < r^2$

Zadanie Znajdź środek i promień koła:

a) $x^2 + y^2 - 2x + 4y - 1 \leq 0$

$(x^2 - 2x) + (y^2 + 4y) - 1 \leq 0$

$\underbrace{(x-1)^2}_{x^2-2x+1} - 1 + \underbrace{(y+2)^2}_{y^2+4y+4} - 4 - 1 \leq 0$

$(x-1)^2 + (y+2)^2 \leq 6$

$S = (1, -2), \quad r = \sqrt{6}$

b) $x^2 + y^2 + 4y \leq 0$

$(x-0)^2 + (y+2)^2 - 4 \leq 0$

$(x-0)^2 + (y+2)^2 \leq 4$

$S = (0, -2) \quad r = 2$

c) $x^2 + y^2 - 8y + 6x < 0$

$\underbrace{x^2 + 6x} + \underbrace{y^2 - 8y} < 0$

$(x+3)^2 - 9 + (y-4)^2 - 16 < 0$

$(x+3)^2 + (y-4)^2 < 25$

$S = (-3, 4), \quad r = 5$

d) $x^2 + y^2 - x + y < 0$

$\underbrace{x^2 - x} + \underbrace{y^2 + y} < 0$

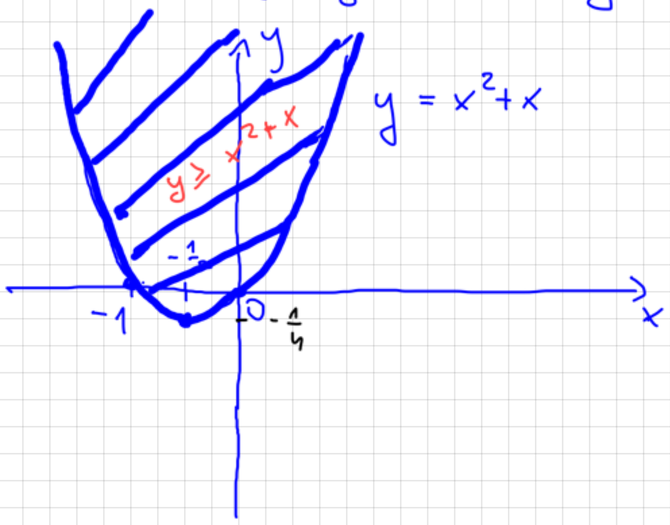
$(x - \frac{1}{2})^2 - \frac{1}{4} + (y + \frac{1}{2})^2 - \frac{1}{4} < 0$

$(x - \frac{1}{2})^2 + (y + \frac{1}{2})^2 < \frac{2}{4}$

$S = (\frac{1}{2}, -\frac{1}{2}) \quad r = \sqrt{\frac{2}{4}} = \frac{\sqrt{2}}{2}$

Zadanie Zaznacz na płaszczyźnie zbiory:

a) $A = \{(x,y) \in \mathbb{R}^2 : y \geq x^2 + x\}$, $B = \{(x,y) \in \mathbb{R}^2 : y - 1 \leq 0\}$



$y \geq x^2 + x$

$y = x^2 + x$ $\quad a = 1,\ b = 1,\ c = 0$

$x^2 + x = 0$ $\quad \Delta = 1$

$x(x+1) = 0$

$x = 0 \ \vee \ x = -1$

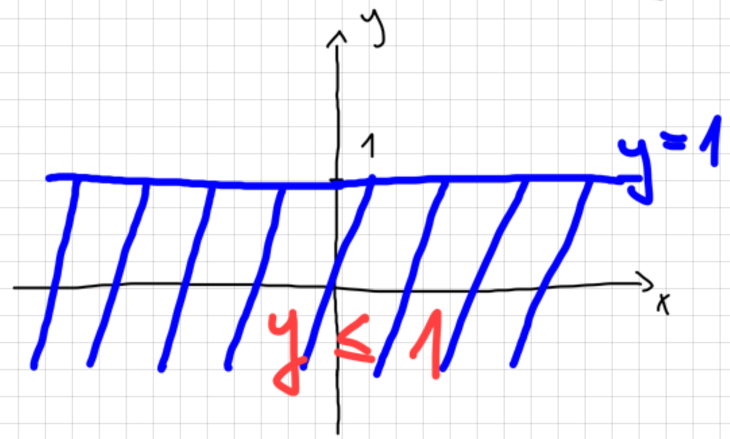
$x_w = -\frac{1}{2}$ $\quad x_w = -\frac{b}{2a} = -\frac{1}{2}$

$y_w = (-\frac{1}{2})^2 - \frac{1}{2} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$ $\quad y_w = -\frac{\Delta}{4a} = -\frac{1}{4}$

$y - 1 \leq 0$

$y \leq 1$

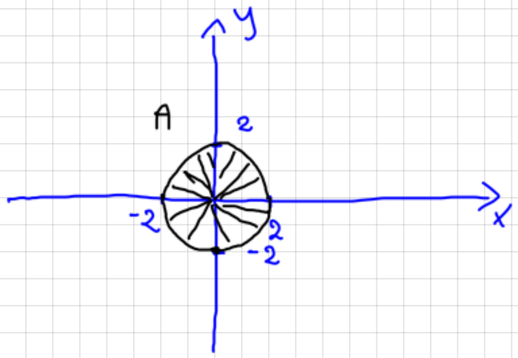
$B = \{(x,y) \in \mathbb{R}^2 : x \in \mathbb{R} \wedge y \leq 1\}$



b) $A = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 4\}$

$x^2 + y^2 \leq 4$

$S = (0,0) \quad r = 2$

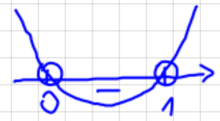


$B = \{(x,y) \in \mathbb{R}^2 : x^2 - x < 0\}$

$x^2 - x < 0$

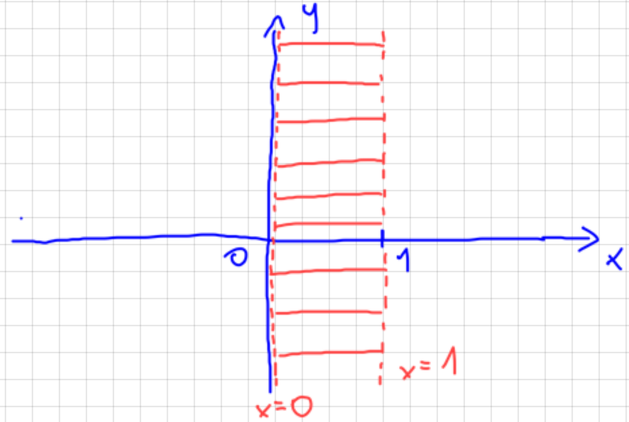
$x(x-1) < 0$

$x = 0 \quad \vee \quad x = 1$



$x \in (0,1)$

$B = \{(x,y) \in \mathbb{R}^2 : x \in (0,1) \wedge y \in \mathbb{R}\}$



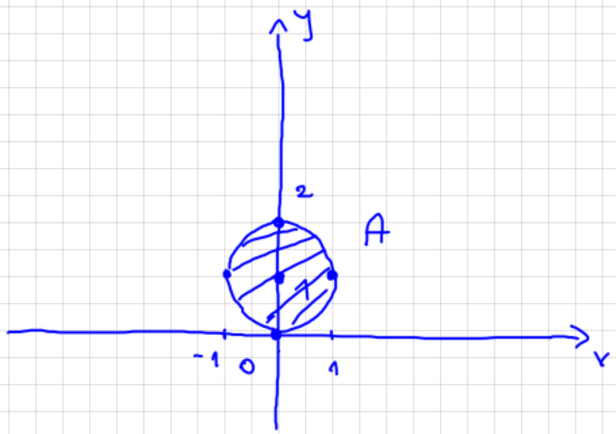
c) $A = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 - 2y \leq 0\}$

$x^2 + y^2 - 2y \leq 0$

$(x-0)^2 + (y-1)^2 - 1 \leq 0$

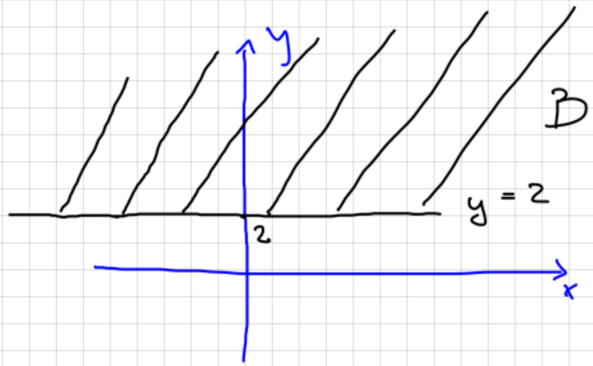
$(x-0)^2 + (y-1)^2 \leq 1$

$S = (0,1) \quad r = 1$



$B = \{(x,y) \in \mathbb{R}^2 : y \geq 2\}$

$y \geq 2$



Zadanie Naszkicuj zbiór $A \cap B$, gdy

a) $A = \{(x,y) \in \mathbb{R}^2 : x^2 + x + y \leq 0\}$

$$x^2 + x + y \leq 0$$
$$y \leq -x^2 - x$$
$$y = -x^2 - x$$
$$-x^2 - x = 0 \qquad a = -1,\ b = -1,\ c = 0$$
$$x(-x - 1) = 0 \qquad \Delta = 1$$
$$x = 0 \quad \vee \quad -x - 1 = 0$$
$$-x = 1$$
$$x = -1$$

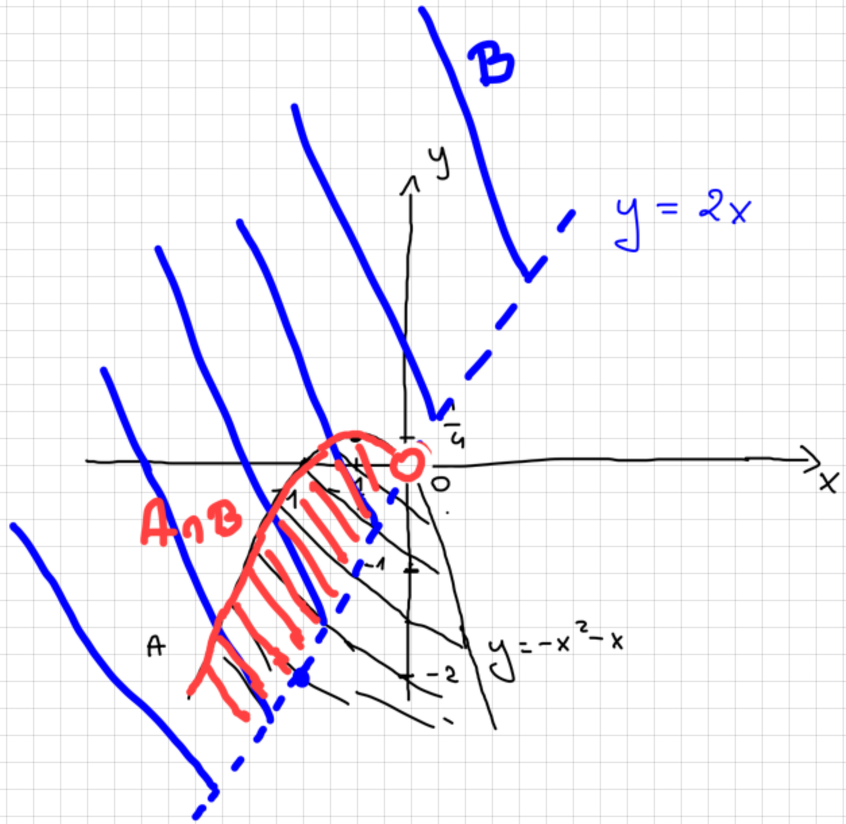
$$x_w = -\frac{b}{2a} = \frac{+1}{-2} = -\frac{1}{2}$$

$$y_w = -\frac{\Delta}{4a} = -\frac{1}{-4} = \frac{1}{4}$$

$B = \{(x,y) \in \mathbb{R}^2 : y - 2x > 0\}$

$$y - 2x > 0$$
$$y > 2x$$
$$y = 2x$$

x	0	-1
$y = 2x$	0	-2



$$A = \{(x,y) \in \mathbb{R}^2 : y \geq x^2 - 2x\}$$

$$B = \{(x,y) \in \mathbb{R}^2 : y + 1 < 0\}$$

$y \geq x^2 - 2x$

$y = x^2 - 2x$

$x^2 - 2x = 0$

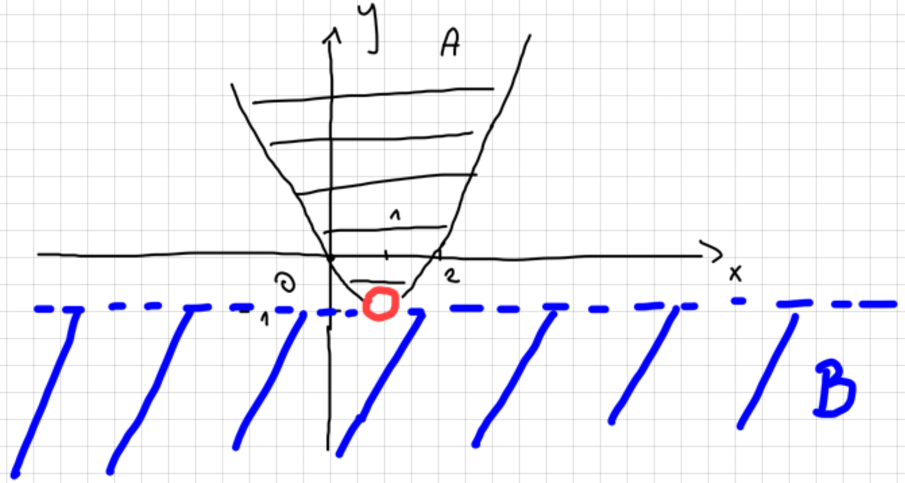
$x(x-2) = 0$

$x = 0 \quad \vee \quad x = 2$

$x_w = 1 \quad , \quad y_w = -1$

$W = (1, -1)$

$W = (1, -1)$



$A \cap B = \emptyset$

Do domu Znajdź $A \cap B$ na płaszczyźnie, gdy

$$A = \{(x,y) \in \mathbb{R}^2 : \quad |x| \geq y \}$$

$$B = \{(x,y) \in \mathbb{R}^2 : \quad y \leq 3 \}$$