$$\int 2x\,dx = x^2 + C,\quad C \in \mathbb{R} \qquad \int 2x\,dt = 2x\int 1\,dt = 2xt + C$$

$$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C \quad \alpha \neq -1$$

$$\int \frac{1}{x}dx = \ln|x| + C$$

1) $\int \frac{1}{\sqrt[3]{x^5}}dx = \int x^{-\frac{5}{3}}dx = \frac{x^{-\frac{2}{3}}}{-\frac{2}{3}} + C = -\frac{3}{2\sqrt[3]{x^2}} + C$

2). $\int \frac{1}{x\ln^3 x}dx = \left\{ \begin{array}{l} \ln x = t \\ \frac{1}{x}dx = dt \end{array} \right\} = \int \frac{1}{t^3}dt = \int t^{-3}dt = \frac{t^{-2}}{-2} + C = -\frac{1}{2t^2} + C = -\frac{1}{2(\ln x)^2} + C$

3). $\int \frac{\cos x}{1+\sin^2 x}dx = \left\{ \begin{array}{l} \sin x = t \\ \cos x\,dx = dt \end{array} \right\} = \int \frac{dt}{1+t^2} = \operatorname{arctg} t + C =$

$= \operatorname{arctg}(\sin x) + C$

4). $\int \frac{x^3}{x^4+5}dx = \left\{ \begin{array}{l} x^4+5 = t \\ 4x^3dx = dt \\ x^3dx = \frac{1}{4}dt \end{array} \right\} = \int \frac{\frac{1}{4}dt}{t} = \frac{1}{4}\int \frac{1}{t}dt = \frac{1}{4}\ln|t| + C =$

$= \frac{1}{4}\ln|x^4+5| + C$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + C$$

5). $\int \frac{x^3}{\sqrt{x^4+5}}dx = \left\{ \begin{array}{l} \sqrt{x^4+5} = t \\ x^4+5 = t^2 \\ 4x^3dx = 2t\,dt \\ x^3dx = \frac{1}{2}t\,dt \end{array} \right\} = \int \frac{\frac{1}{2}\cancel{t}\,dt}{\cancel{t}} = \frac{1}{2}\int 1\,dt = \frac{1}{2}t + C =$

$= \frac{1}{2}\sqrt{x^4+5} + C$

$$\int \frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)} + C$$

$\frac{1}{4}\int \frac{4x^3}{\sqrt{x^4+5}}dx = \frac{1}{4}\cdot 2\sqrt{x^4+5} + C$

6). $\int e^{3x}dx = \frac{e^{3x}}{3} + C \qquad \int e^{ax+b}dx = \frac{e^{ax+b}}{a} + C$

7) $\frac{1}{2}\int \frac{1\cdot 2}{2x+3}dx = \frac{1}{2}\ln|2x+3| + C$

$$\boxed{\int f\cdot g' = f\cdot g - \int f'g}$$

8). $\int \cos(x+3)dx = \sin(x+3) + C$

9). $\int \underbrace{x}_{f}\underbrace{\cos x}_{g'}dx = \left\{ \begin{array}{ll} f(x) = x & g'(x) = \cos x \\ f'(x) = 1 & g(x) = \sin x \end{array} \right\} = x\sin x - \int \sin x + C = x\sin x + \cos x + C$

10). $\int \underbrace{x}_{g'}\underbrace{\operatorname{arctg} x}_{f}dx = \left\{ \begin{array}{ll} f(x) = \operatorname{arctg} x & g'(x) = x \\ f'(x) = \frac{1}{1+x^2} & g(x) = \frac{x^2}{2} \end{array} \right\} = \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2}\int \frac{x^2}{x^2+1}dx =$

$= \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2}\int \frac{x^2+1-1}{x^2+1}dx = \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2}\int \left(1 - \frac{1}{x^2+1}\right)dx =$

$$= \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2}(x - \operatorname{arctg} x) + C$$

11). $\int \underbrace{1}_{g'} \underbrace{\ln 3x}_{f}\, dx = \left\{ \begin{matrix} f(x) = \ln 3x & g'(x) = 1 \\ f'(x) = \frac{1}{3x}\cdot 3 & g(x) = x \end{matrix} \right\} = x\ln 3x - \int 1\, dx = x\ln 3x - x + C$

III. proste

$$\frac{A}{(x-a)^n} \qquad \frac{Ax+B}{(x^2+px+q)^n} \quad \Delta<0$$

podstawienie

$$\boxed{\int \frac{1}{x^2+k}\, dx \overset{\frac{x}{\sqrt{k}}=t}{=} \frac{1}{\sqrt{k}}\operatorname{arctg}\frac{x}{\sqrt{k}} + C \quad k>0}$$

$$\int \frac{1}{x^2+x+1}\, dx = \int \frac{1}{\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}}\, dx = \frac{1}{\sqrt{\frac{3}{4}}}\operatorname{arctg}\frac{x+\frac{1}{2}}{\sqrt{\frac{3}{4}}} + C$$

$\Delta<0$

pochodna

$$\int \frac{x}{x^2+x+1}\, dx = \frac{1}{2}\int \frac{2x+1}{x^2+x+1}\, dx - \frac{1}{2}\int \frac{1}{x^2+x+1}\, dx$$

$\ln|x^2+x+1|$

$$x = \frac{1}{2}(2x+1) - \frac{1}{2}$$

$\int \frac{1}{(x-1)(x^2+3)}\, dx \to$ rozkład na ułamki proste

$$= \int \frac{A}{x-1}\, dx + \int \frac{Bx+C}{x^2+3}\, dx$$

$$A(x^2+3) + (Bx+C)(x-1) = 1$$

x^2: $A+B=0$

x: $C-B=0$ $\implies$ $A = \dots$, $B = \dots$, $C = \dots$

x^0: $3A-C=1$