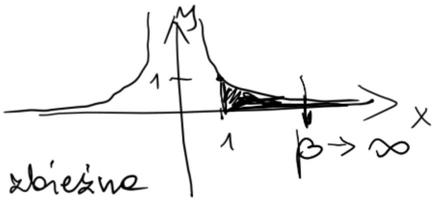
$$\int_1^\infty \frac{1}{x^6}dx = \lim_{\beta\to\infty}\int_1^\beta \frac{1}{x^6}dx = \lim_{\beta\to\infty}\int_1^\beta x^{-6}dx =$$

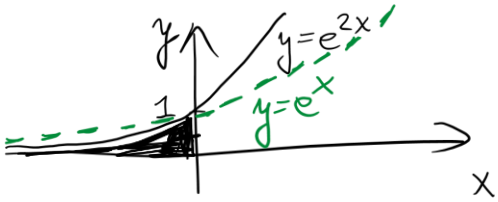
$$= \lim_{\beta\to\infty}\frac{x^{-5}}{-5}\Big|_1^\beta = \lim_{\beta\to\infty}\frac{1}{-5x^5}\Big|_1^\beta = \lim_{\beta\to\infty}\Big(\underbrace{-\frac{1}{5\beta^5}}_{\to 0}+\frac{1}{5}\Big) = \frac{1}{5}$$

całka zbieżna



$$\int_{-\infty}^0 e^{2x}dx = \lim_{\alpha\to-\infty}\int_\alpha^0 e^{2x}dx = \lim_{\alpha\to-\infty}\frac{e^{2x}}{2}\Big|_\alpha^0 =$$

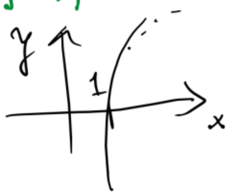
$$= \lim_{\alpha\to-\infty}\Big(\frac{e^0}{2} - \underbrace{\frac{e^{2\alpha}}{2}}_{\to 0}\Big) = \frac{1}{2}$$



$$\int_2^\infty \frac{x^2}{x^3+1}dx = \lim_{\beta\to\infty}\frac{1}{3}\int_2^\beta \frac{3x^2}{x^3+1}dx =$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)|+C$$

$$= \lim_{\beta\to\infty}\frac{1}{3}\ln|x^3+1|\Big|_2^\beta = \lim_{\beta\to\infty}\Big(\frac{1}{3}\ln|\underbrace{\beta^3+1}_{\to\infty}| - \frac{1}{3}\ln|2^3+1|\Big) = \infty$$



$$\int_{-\infty}^0 xe^x dx = \lim_{\alpha\to-\infty}\int_\alpha^0 xe^x dx = (*)$$

$$\int xe^x dx = \left\{\begin{matrix} f(x)=x & g'(x)=e^x \\ f'(x)=1 & g(x)=e^x\end{matrix}\right\} = xe^x - \int e^x dx = xe^x - e^x + C$$

$$(*) = \lim_{\alpha\to-\infty}\left(xe^x - e^x\right)\Big|_\alpha^0 = \lim_{\alpha\to-\infty}\left(0 - e^0 - (\alpha e^\alpha - e^\alpha)\right) =$$

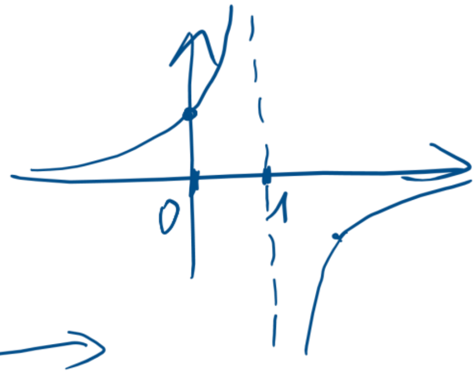
$$= \lim_{\alpha\to-\infty}\big(-1 - \underbrace{\alpha e^\alpha}_{0} + \underbrace{e^\alpha}_{\to 0}\big) = -1$$

$$\lim_{\alpha\to-\infty}\alpha e^\alpha = [-\infty\cdot 0] = \lim_{\alpha\to-\infty}\frac{\alpha}{e^{-\alpha}} = \left[\frac{-\infty}{\infty}\right] \overset{H}{=} \lim_{\alpha\to-\infty}\frac{1}{e^{-\alpha}(-1)} = \frac{1}{-\infty} = 0$$

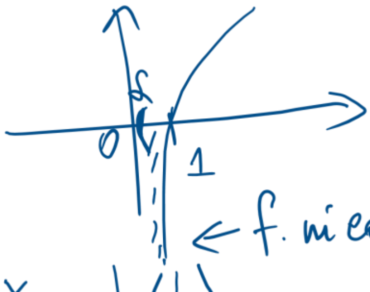
$$\int_{-\infty}^\infty \frac{1}{9+x^2}dx = \int_{-\infty}^0 \frac{1}{9+x^2}dx + \int_0^\infty \frac{1}{9+x^2}dx$$

$$\int_0^1 \frac{1}{1-x}dx = -\lim_{\beta\to1^-}\int_0^\beta \frac{-1}{1-x}dx = -\lim_{\beta\to1^-}\ln|1-x|\Big|_0^\beta =$$

$$= -\lim_{\beta\to1^-}\ln|\underbrace{1-\beta}_{\to 0^+}| - \ln 1 = +\infty$$



$$\int_{\boxed{0}}^1 \ln x\,dx = \lim_{\alpha\to0^+}\int_\alpha^1 \ln x\,dx \ \ldots$$



← f. nieograniczona

$$\int_1^3 \frac{1}{(x-2)^2}dx = \int_1^{\boxed{2}} \frac{1}{(x-2)^2}dx + \int_{\boxed{2}}^3 \frac{1}{(x-2)^2}dx$$

$$\int_1^3 \frac{dx}{\sqrt{(x-1)(3-x)}} = \int_{\boxed{1}}^c \frac{dx}{\sqrt{(x-1)(3-x)}} + \int_c^{\boxed{3}} \frac{dx}{\sqrt{(x-1)(3-x)}}$$

→ postać kanoniczna

$$\int \frac{1}{\sqrt{k-x^2}}dx = \arcsin\frac{x}{\sqrt{k}} + C \quad k>0$$

$$\int \frac{1}{\sqrt{k+x^2}}dx = \ln\left|x+\sqrt{k+x^2}\right| + C$$