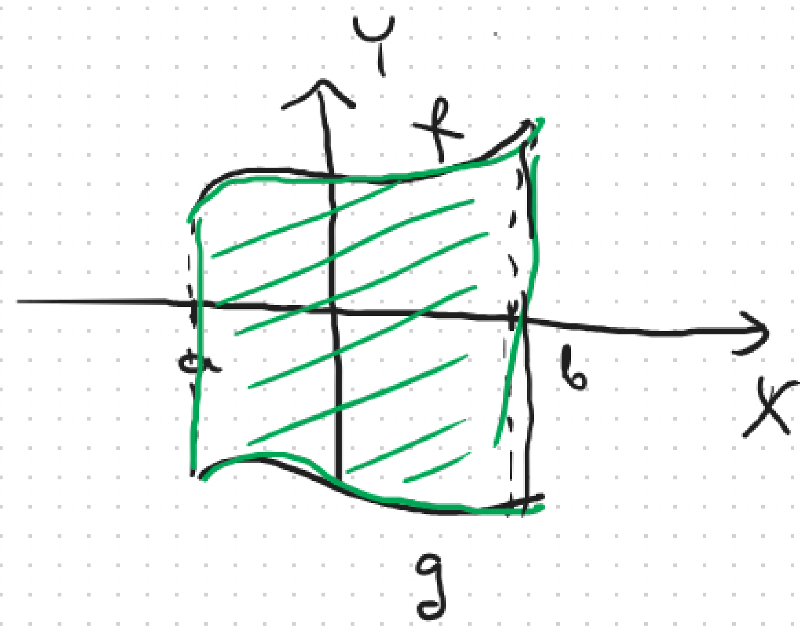
Tw. Newtona - Leibniza. Jeżeli $f:[a,b]\to\mathbb{R}$ jest ciągła i F jest dowolną funkcją pierwotną funkcji f, to

$$\int_a^b f(x)dx = F(b)-F(a) \qquad \Big\} \; F(b)-F(a) = \Big[F(x)\Big]\Big|_a^b$$

$$\int_{-1}^{\sqrt3} \frac{1}{1+x^2}dx = \Big[\operatorname{arctg} x\Big]\Big|_{-1}^{\sqrt3} = \operatorname{arctg}\sqrt3 - \operatorname{arctg}(-1) =$$

$$= \frac{\pi}{3} - \left(-\frac{\pi}{4}\right) = \frac{7}{12}\pi$$

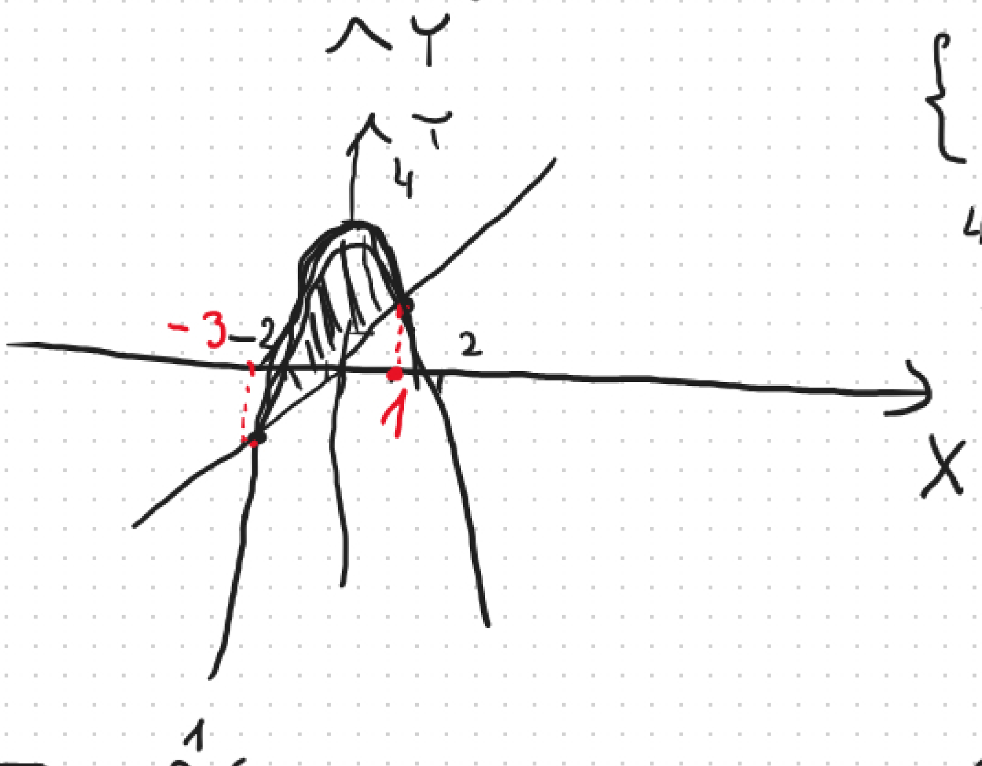
$$\int \frac{1}{1+x^2}dx = \operatorname{arctg}x + C$$



$$P = \int_a^b \left(f(x)-g(x)\right)dx$$

Obliczyć pole obszaru ograniczonego krzywymi

① $y = 4-x^2$, $y = 2x+1$



$$\begin{cases} y = 4-x^2 \\ y = 2x+1 \end{cases}$$

$4-x^2 = 2x+1$

$-x^2-2x+3=0$

$\Delta = 4+12 = 16$

$x_1 = \frac{2+4}{-2} = -3$

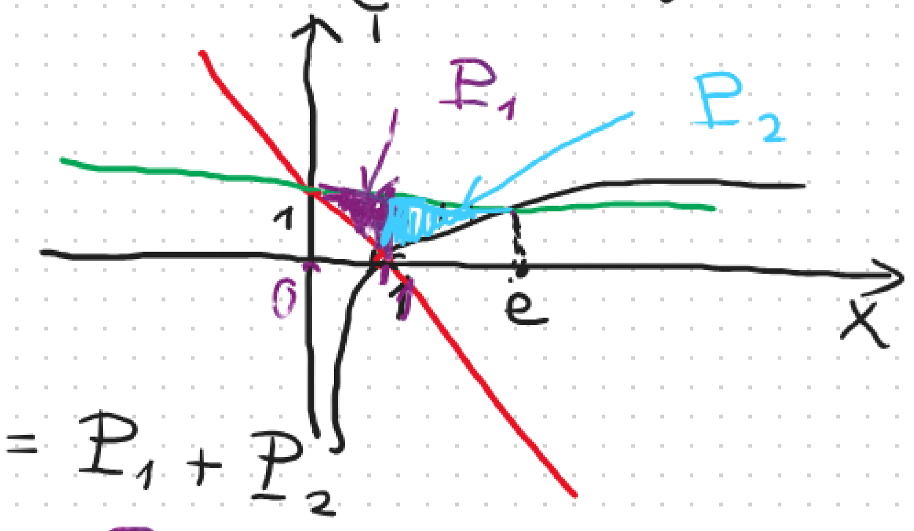
$x_2 = \frac{2-4}{-2} = 1$

$$P = \int_{-3}^{1}\left(4-x^2-(2x+1)\right)dx = \int_{-3}^{1}(3-2x-x^2)dx =$$

$$= \left[3x-x^2-\frac{x^3}{3}\right]\Big|_{-3}^{1} = 3-1-\frac13-(-9-9+9) =$$

$$= 2-\frac13+9 = \frac{32}{3}$$

ⓑ $y = \ln x$, $y = 1-x$, $y = 1$



$$\begin{cases} y = 1 \\ y = \ln x \end{cases}$$

$\ln x = 1$

$x = e^1 = e$

$P = P_1 + P_2$

$P_1 = P_\Delta = \frac12 \cdot 1\cdot 1 = \frac12$

$$P_1 = \int_0^1 \left(1-(1-x)\right)dx = \int_0^1 x\,dx = \left[\frac{x^2}{2}\right]\Big|_0^1 = \frac12 - \frac{0^2}{2} = \frac12$$

$$P_2 = \int_1^e (1-\ln x)\,dx = \left[x-(x\ln x - x)\right]\Big|_1^e =$$

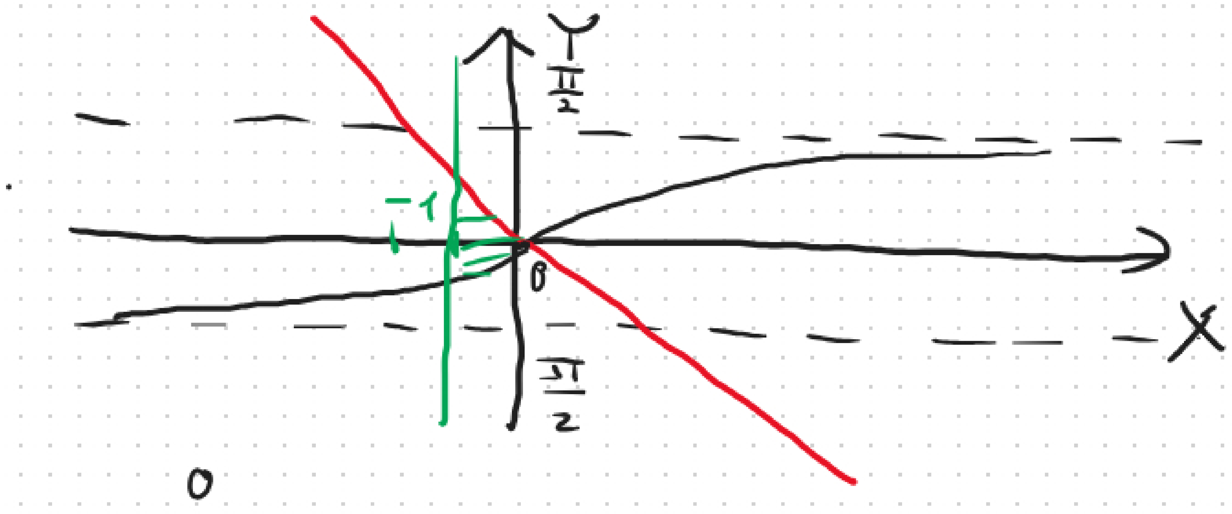
$$\int 1\cdot\ln x\,dx = \left\{ \begin{matrix} f(x)=\ln x & g'(x)=1 \\ f'(x)=\frac1x & g(x)=x \end{matrix}\right\} = x\cdot\ln x - \int 1\,dx =$$

$$= \boxed{x\cdot\ln x - x} + C \qquad \int f g' = f g - \int f' g$$

$$= \left[2x - x\ln x\right]\Big|_1^e = 2e - e\cdot 1 - (2\cdot 1 - 1\cdot 0) = e-2$$

$$P = P_1 + P_2 = \frac12 + e - 2 = e - \frac32$$

(c) $y = \operatorname{arctg}x$, $y=-x$, $x=-1$



$$P = \int_{-1}^{0}\left(-x-\operatorname{arctg}x\right)dx = \left[-\frac{x^2}{2} - x\cdot\operatorname{arctg}x + \frac12\ln(1+x^2)\right]\Big|_{-1}^{0} =$$

$$= \frac12 - \underbrace{\operatorname{arctg}(-1)}_{-\frac{\pi}{4}} - \frac12\ln 2 = \frac12 + \frac{\pi}{4} - \frac12\ln 2$$

$$\int \operatorname{arctg}x\,dx = \left\{ \begin{matrix} f(x)=\operatorname{arctg}x & g'(x)=1 \\ f'(x)=\frac{1}{1+x^2} & g(x)=x \end{matrix}\right\} =$$

$$= x\cdot\operatorname{arctg}x - \frac12\int\frac{2x}{1+x^2}dx = x\cdot\operatorname{arctg}x - \frac12\ln|1+x^2| + C$$

$$\bigwedge_{x\in\mathbb{R}} 1+x^2>0$$