Całki nieoznaczone – 20.01.25

$t = 3-x$

1) $\int \ln^2 x\,dx$ $\boxed{2}$ $\int \frac{x}{(3-x)^5}dx$ 3) $\int \frac{e^{2x}}{\sqrt{4-e^{2x}}}dx$ 4) $\int \frac{e^x}{\sqrt{4-e^{2x}}}dx$ 5) $\int \sqrt{x}\ln 3x\,dx$

$t=\sin x$ $t=\cos x$

6) $\int \sin x\cos x\,dx$ lub $t=\cos x$ 7) $\int \sin x\cos^2 x\,dx$ 8) $\int \frac{1}{x(\ln^2 x+3)}dx$ 9) $\int (3x-1)e^{2x}dx$

$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}+C$

$t=\arcsin x$

$\boxed{10}$ $\int \frac{5-3x}{x^2-3x+2}dx$ $\boxed{11}$ $\int \frac{5-3x}{x^2-2x+3}dx$ 12) $\int \sqrt{\frac{\arcsin x}{1-x^2}}dx$ 13) $\int \frac{x}{\cos^2 x}dx$

$\Delta = ?$

Całkowanie przez części

$fg - \int f'g$

9) $\int (3x-1)\cdot e^{2x}dx = \left\{\begin{matrix} f(x)=3x-1 & g'(x)=e^{2x} \\ f'(x)=3 & g(x)=\frac{1}{2}e^{2x}\end{matrix}\right\} = (3x-1)\cdot\frac{1}{2}e^{2x} - \int \frac{3}{2}e^{2x}dx =$

$= (3x-1)\cdot\frac{1}{2}e^{2x} - \frac{3}{2}\cdot\frac{1}{2}e^{2x}+C$

5) $\int \sqrt{x}\cdot\ln 3x\,dx = \left\{\begin{matrix} f(x)=\ln 3x & g'(x)=x^{\frac{1}{2}} \\ f'(x)=\cancel{3}\cdot\frac{1}{\cancel{3}x} & g(x)=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\end{matrix}\right\} = \frac{2}{3}x^{\frac{3}{2}}\ln 3x - \int \frac{2}{3}\cdot\boxed{\frac{1}{x}\cdot x^{\frac{3}{2}}}dx =$

$x^{\frac{1}{2}}$

$= \frac{2}{3}x^{\frac{3}{2}}\ln 3x - \frac{2}{3}\cdot\frac{2}{3}\cdot x^{\frac{3}{2}}+C$

13) $\int \frac{x}{\cos^2 x}dx = \int x\cdot\frac{1}{\cos^2 x}dx = \left\{\begin{matrix} f(x)=x & g'(x)=\frac{1}{\cos^2 x} \\ f'(x)=1 & g(x)=\operatorname{tg} x\end{matrix}\right\} = x\operatorname{tg}x - \int \operatorname{tg}x\,dx =$

$= x\operatorname{tg}x + \int \frac{-\sin x}{\cos x}dx =$

$\int \frac{f'}{f} = \ln|f|$

$= x\operatorname{tg}x + \ln|\cos x| + C$

1) $\int \ln^2 x\,dx = \left\{\begin{matrix} f(x)=\ln^2 x & g'(x)=1 \\ f'(x)=2\ln x\cdot\frac{1}{x} & g(x)=x\end{matrix}\right\} = x\ln^2 x - \int 2\ln x\cdot\frac{1}{\cancel{x}}\cdot x\,dx =$

$\left\{\begin{matrix} f(x)=\ln x & g'(x)=2\end{matrix}\right\}$

$= x\ln^2 x - (\ldots)$ do dokończenia

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$\Delta = ?$

Całkowanie przez podstawienie

8) $\int \frac{1}{x(\ln^2 x+3)}dx = \int \frac{1}{x}\cdot\frac{1}{\ln^2 x+3}dx = \left\{\begin{matrix} t=\ln x \\ dt=\frac{1}{x}dx\end{matrix}\right\} = \int \frac{1}{t^2+3}dt =$

$= \frac{1}{\sqrt{3}}\operatorname{arctg}\frac{t}{\sqrt{3}}+C = \frac{1}{\sqrt{3}}\operatorname{arctg}\frac{\ln x}{\sqrt{3}}+C$

$k=3$

3) $\int \frac{e^x}{\sqrt{4-e^{2x}}}dx = \left\{\begin{matrix} t=e^x \\ dt=e^x dx\end{matrix}\right\} = \int \frac{dt}{\sqrt{4-t^2}} = \arcsin\frac{t}{2}+C = \arcsin\frac{e^x}{2}+C$

4) $\frac{1}{2}\int \frac{2e^{2x}}{\sqrt{4-e^{2x}}}dx = \left\{\begin{matrix} t=e^{2x} \\ dt=2e^{2x}dx\end{matrix}\right\} = \frac{1}{2}\int \frac{dt}{\sqrt{4-t}} = \left\{\begin{matrix} u=4-t \\ du=-dt\end{matrix}\right\} =$

$= -\frac{1}{2}\int \frac{1}{\sqrt{u}}du = -\int \frac{1}{2\sqrt{u}}du = -\sqrt{u}+C =$

$= -\sqrt{4-t}+C = -\sqrt{4-e^{2x}}+C$

lub $t = 4-e^{2x}$ lub wzór $\int \frac{f'}{\sqrt{f}}$

7) $-\int -\sin x\cos^2 x\,dx = \left\{\begin{matrix} t=\cos x \\ dt=-\sin x\,dx\end{matrix}\right\} = -\int t^2 dt = -\frac{t^3}{3}+C$

$= -\frac{\cos^3 x}{3}+C$

Całkowanie wybranych funkcji wymiernych

10) $\Delta = 9-4\cdot 2 = 1 > 0 \Rightarrow x^2-3x+2 = (x-1)(x-2)$

$\frac{5-3x}{x^2-3x+2} = \frac{5-3x}{(x-1)(x-2)} = \left\{\begin{matrix}\text{rozkład na} \\ \text{ułamki proste}\end{matrix}\right\} = \frac{A}{x-1}+\frac{B}{x-2} \equiv \quad A, B = ?$

$= \frac{A(x-2)+B(x-1)}{(x-1)(x-2)}$

$5-3x = A(x-2)+B(x-1)$ dla $x\in\mathbb{R}$

$x=2$ $\quad 5-6 = B(2-1) \Rightarrow B = \frac{-1}{1} = -1$

$x=1$ $\quad 5-3 = A(1-2) \Rightarrow A = \frac{2}{-1} = -2$

$\int \frac{5-3x}{x^2-3x+2}dx = \int\left(\frac{-2}{x-1}+\frac{-1}{x-2}\right)dx = -2\int \frac{1}{x-1}dx - \int \frac{1}{x-2}dx = -2\ln|x-1| - \ln|x-2| + C$

11) $\int \frac{5-3x}{x^2-2x+3}dx = \boxed{-\frac{3}{2}}\int \frac{2x-2}{x^2-2x+3}dx + \boxed{2}\int \frac{1}{x^2-2x+3}dx$

$\Delta = 4-4\cdot 3 = 4-12 < 0$

$\left\{-\frac{3}{2}\cdot(2x-2)+2 = 5-3x\right\}$

$= -\frac{3}{2}\ln|x^2-2x+3| + 2\,(*)$

$\int \frac{1}{x^2+k}dx = \frac{1}{\sqrt{k}}\operatorname{arctg}\frac{x}{\sqrt{k}}$

$(*) = \int \frac{1}{x^2-2x+3}dx = \int \frac{1}{(x^2-2x+1)+2}dx = \int \frac{1}{(x-1)^2+2}dx = \left\{\begin{matrix} t=x-1 \\ dt=dx\end{matrix}\right\} =$

$= \int \frac{1}{t^2+2}dt \overset{k=2}{=} \frac{1}{\sqrt{2}}\operatorname{arctg}\frac{t}{\sqrt{2}}+C = \frac{1}{\sqrt{2}}\operatorname{arctg}\frac{x-1}{\sqrt{2}}+C$