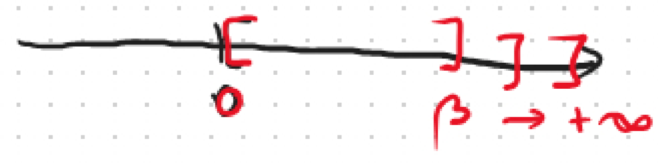
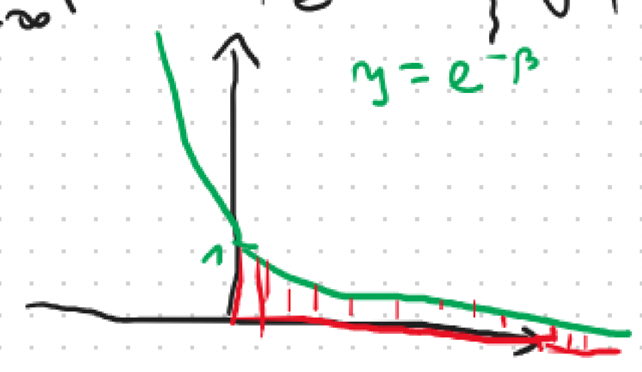
① $\int_0^{+\infty} e^{-x}dx = \lim_{\beta\to+\infty}\int_0^{\beta} e^{-x}dx = \lim_{\beta\to+\infty}\left[-e^{-x}\right]\Big|_0^{\beta} =$

$\int e^{-x}dx = \dots$



$= \lim_{\beta\to+\infty}\left(-e^{-\beta}+e^{0}\right) = \{0+1\} = 1$ – CAŁKA ZBIEŻNA



② $\int_{-\infty}^{1} \frac{1}{x^2-4x+5}dx = \lim_{\alpha\to-\infty}\int_{\alpha}^{1}\frac{1}{x^2-4x+5}dx = (*)$

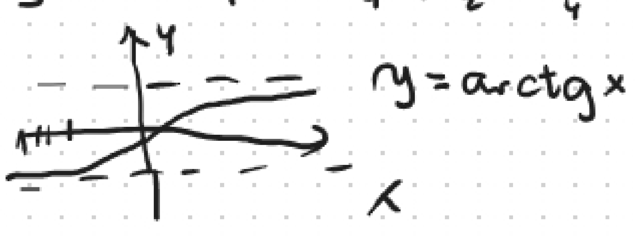
$\Delta<0$

$\int \frac{1}{x^2-4x+5}dx = \int \frac{1}{(x^2-4x+4)+1}dx = \int \frac{1}{(x-2)^2+1}dx = \left\{\begin{matrix} x-2=t \\ dx=dt\end{matrix}\right\} =$

$= \int \frac{1}{t^2+1}dt = \operatorname{arctg} t + C = \operatorname{arctg}(x-2)+C$

$(*) = \lim_{\alpha\to-\infty}\left[\operatorname{arctg}(x-2)\right]\Big|_{\alpha}^{1} = \lim_{\alpha\to-\infty}\left(\operatorname{arctg}(-1)-\operatorname{arctg}(\alpha-2)\right) =$

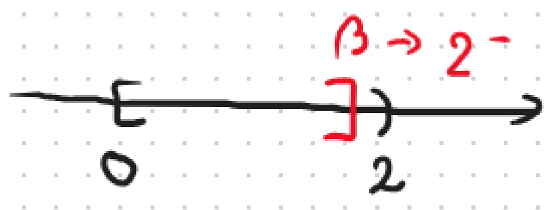
$= \lim_{\alpha\to-\infty}\left(-\frac{\pi}{4}-\operatorname{arctg}(\alpha-2)\right) = \left\{-\frac{\pi}{4}-\operatorname{arctg}(-\infty)\right\} = -\frac{\pi}{4}+\frac{\pi}{2} = \frac{\pi}{4}$



③ $\int_{-\infty}^{+\infty}\frac{1}{x^2-4x+5}dx = \int_{-\infty}^{1}\frac{1}{x^2-4x+5}dx + \int_{1}^{+\infty}\frac{1}{x^2-4x+5}dx$

④ $\int_0^2 \frac{x}{\sqrt{4-x^2}}dx = \lim_{\beta\to2^-} -\frac{1}{2}\int_0^{\beta}\frac{-2x}{\sqrt{4-x^2}}dx =$

$D: 4-x^2>0$
$D = (-2,2)$



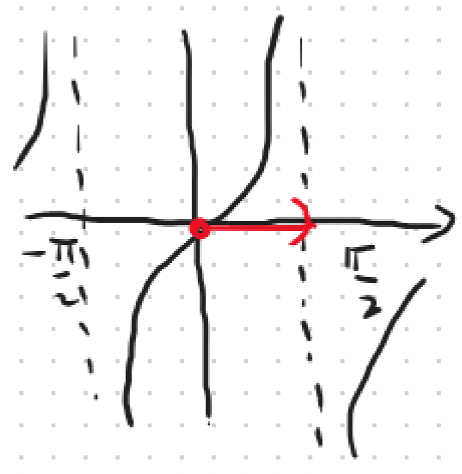
$= \lim_{\beta\to2^-}\left[-\frac{1}{2}\cdot2\sqrt{4-x^2}\right]\Big|_0^{\beta} =$

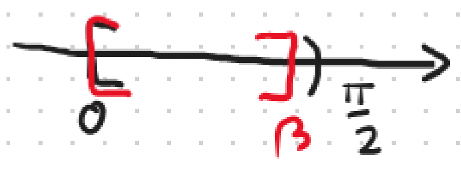
$\int\frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)}+C$

$f(x)>0$

$= \lim_{\beta\to2^-}\left[-\sqrt{4-\beta^2}+2\right] = 0+2 = 2$

⑤ $\int_0^{\frac{\pi}{2}}\operatorname{tg}x\,dx = \lim_{\beta\to\frac{\pi}{2}^-}\int_0^{\beta}\operatorname{tg}x\,dx = (*)$





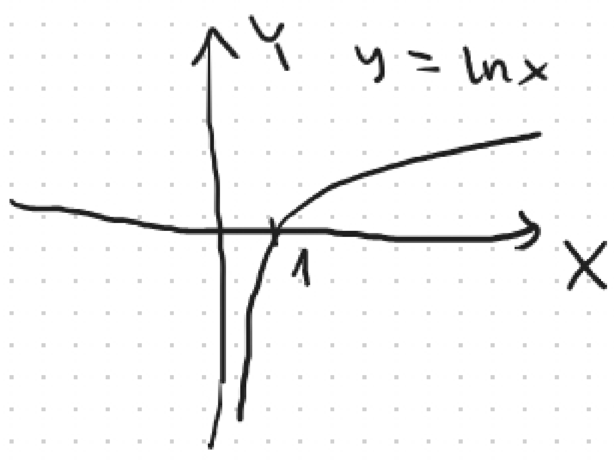
$\int\operatorname{tg}x\,dx = -\int\frac{-\sin x}{\cos x}dx = -\ln|\cos x|+C$

$\int\frac{f'(x)}{f(x)}dx = \ln|f(x)|+C$

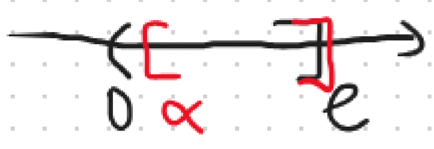
$(*)\ \lim_{\beta\to\frac{\pi}{2}^-}\left[-\ln|\cos x|\right]\Big|_0^{\beta} = \lim_{\beta\to\frac{\pi}{2}^-}\left[-\ln|\cos\beta| + \underbrace{\ln|\cos 0|}_{=0}\right] =$

$= \{-\ln(0^+)\} = -(-\infty) = +\infty$

$\cos 0 = 1$ $\quad \cos\frac{\pi}{2} = 0$
$\ln 1 = 0$



(6) $\int_0^e \ln x\,dx = \lim_{\alpha\to0^+}\int_{\alpha}^{e}\ln x\,dx = (*)$



$\int\ln x\,dx = \dots = x\ln x - x + C = x(\ln x - 1)+C$

$(*) = \lim_{\alpha\to0^+}\left[x(\ln x-1)\right]\Big|_{\alpha}^{e} = \lim_{\alpha\to0^+}\left(0-\alpha(\ln\alpha-1)\right) =$

$= \{0\cdot(-\infty)\} = \lim_{\alpha\to0^+}\frac{-\ln\alpha+1}{\frac{1}{\alpha}} = \left\{\frac{+\infty}{+\infty}\right\} \overset{H}{=} \lim_{\alpha\to0^+}\frac{-\frac{1}{\alpha}}{-\frac{1}{\alpha^2}} =$

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$= \lim_{\alpha\to0^+}\left(\frac{1}{\alpha}\cdot\alpha^2\right) = \lim_{\alpha\to0^+}\alpha = 0$

(7) $\int_1^3\frac{x}{4-x^2}dx = \int_1^{2}\frac{x}{4-x^2}dx + \int_{2}^{3}\frac{x}{4-x^2}$

$D = \mathbb{R}\setminus\{2,-2\}$

(8) $\int_0^{+\infty}\ln x\,dx = \int_{0}^{e}\ln x\,dx + \int_e^{+\infty}\ln x\,dx =$

(9) $\int_{-1}^{0}\frac{1}{x^2}e^{\frac{1}{x}}dx = \dots$

$D = \mathbb{R}\setminus\{0\}$