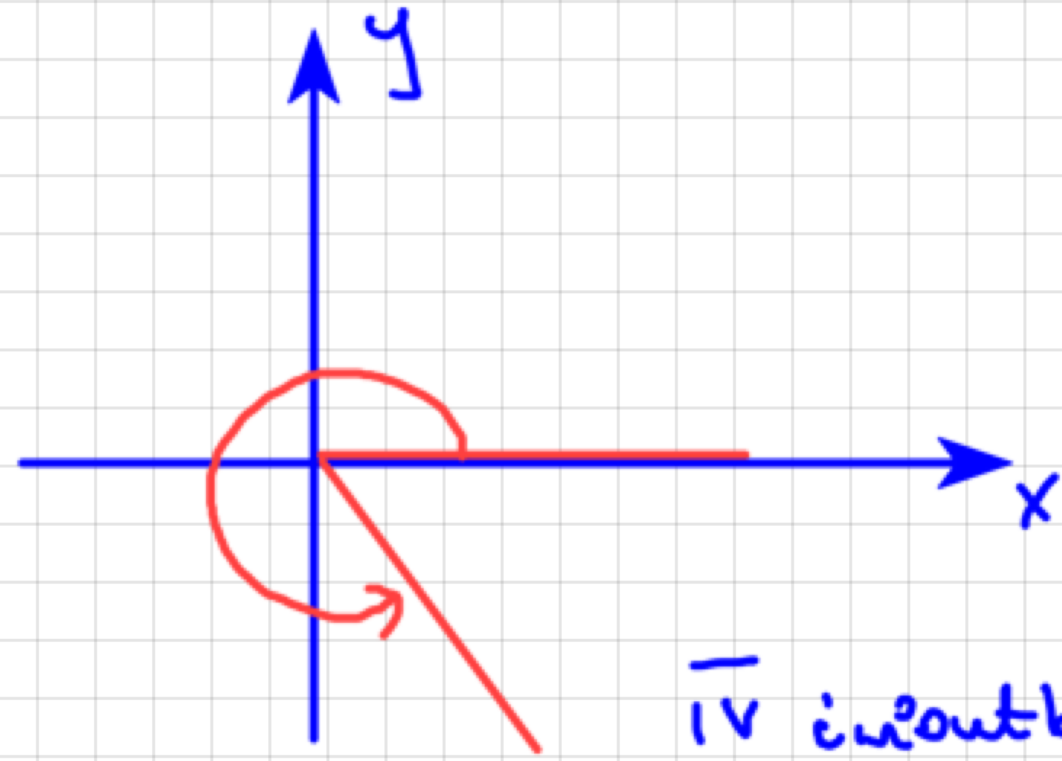
27.10.2025 r.

Zadanie Oblicz

nieparzysta krotność 90°

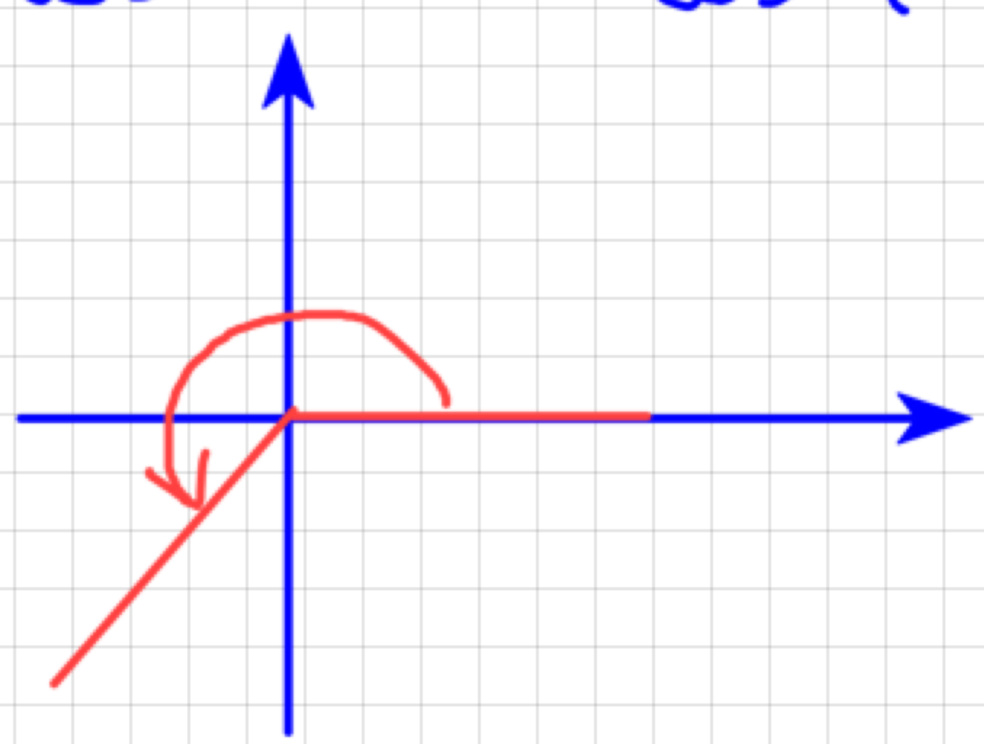
$$\sin 300^\circ = \sin(3 \cdot 90^\circ + 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$$



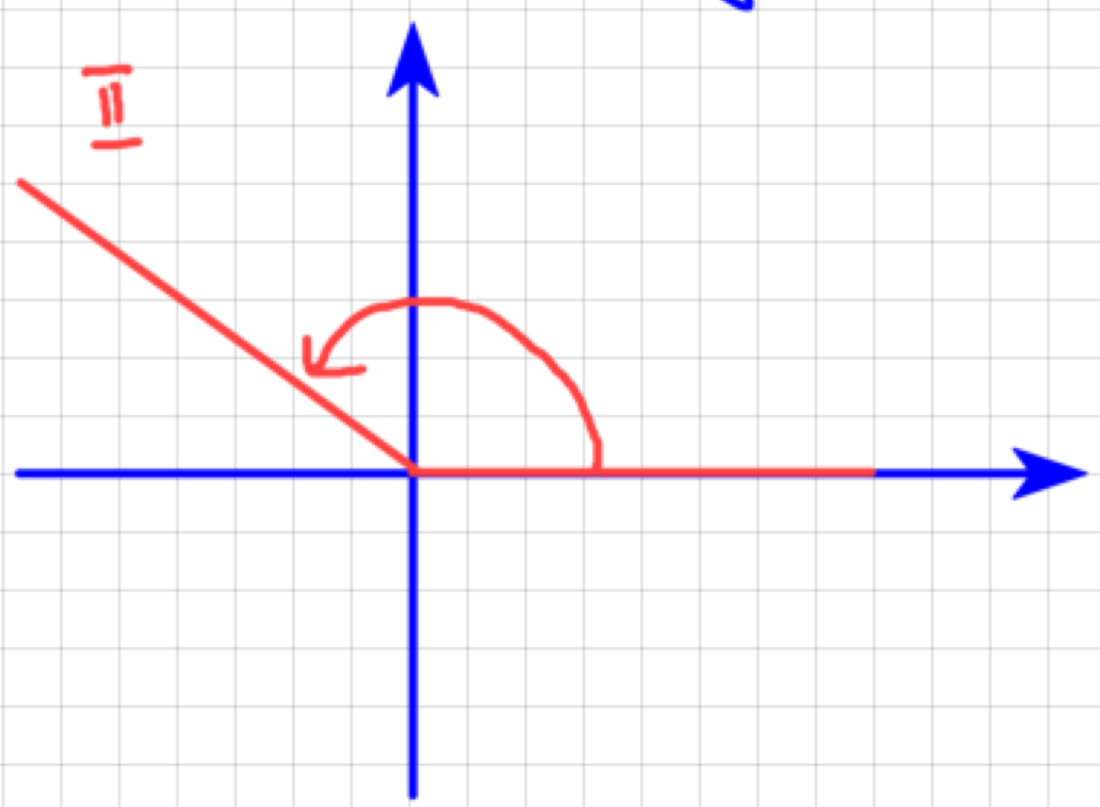
IV ćwiartka

krotność 90° jest parzysta

$$\cos 225^\circ = \cos(2 \cdot 90^\circ + 45^\circ) = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$$



$$\operatorname{tg} 150^\circ = \operatorname{tg}(1 \cdot 90^\circ + 60^\circ) = -\operatorname{ctg} 60^\circ = -\frac{\sqrt{3}}{3}$$

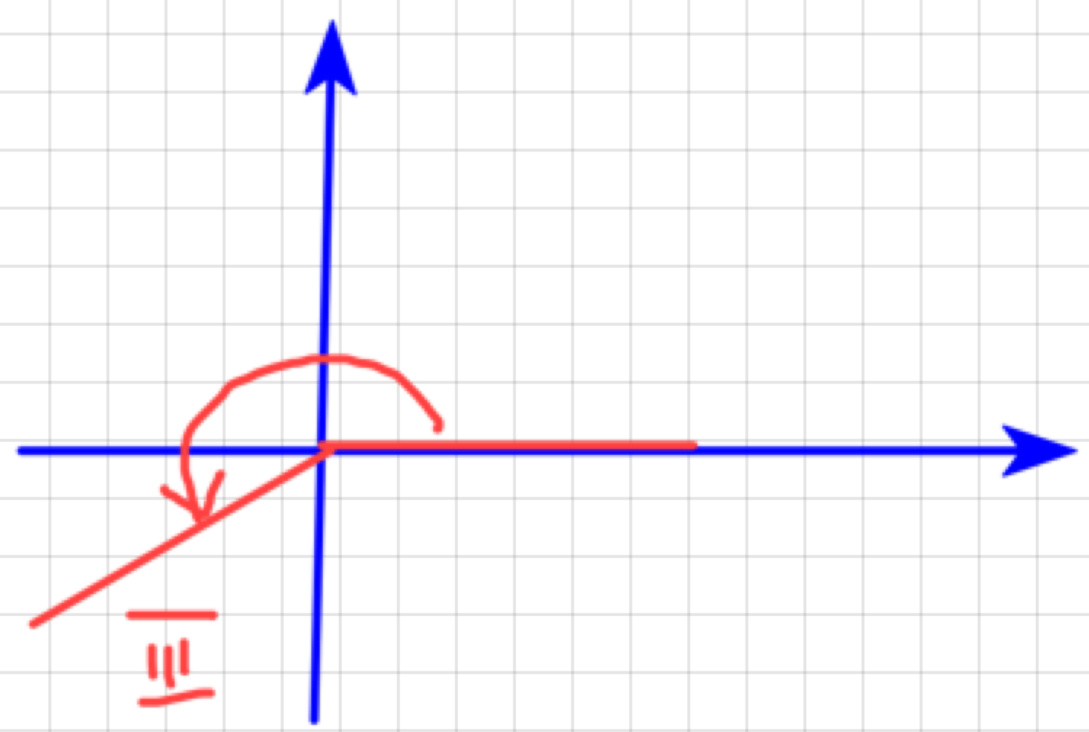


$$\cos 60^\circ = \frac{1}{2}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\operatorname{ctg} 60^\circ = \frac{\cos 60^\circ}{\sin 60^\circ} = \frac{\frac{1}{\cancel{2}_1}}{\frac{\sqrt{3}}{\cancel{2}_1}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\operatorname{ctg} 210^\circ = \operatorname{ctg}(2 \cdot 90^\circ + 30^\circ) = +\operatorname{ctg} 30^\circ = \sqrt{3}$$



$$\operatorname{ctg} 30^\circ = \frac{\cos 30^\circ}{\sin 30^\circ} = \frac{\frac{\sqrt{3}}{\cancel{2}_1}}{\frac{1}{\cancel{2}_1}} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

Zbiory na płaszczyźnie i wykresy

Iloczyn kartezjański zbiorów A i B to zbiór :

$$A \times B = \{ (x,y) : \ x \in A \ \wedge \ x \in B \}.$$

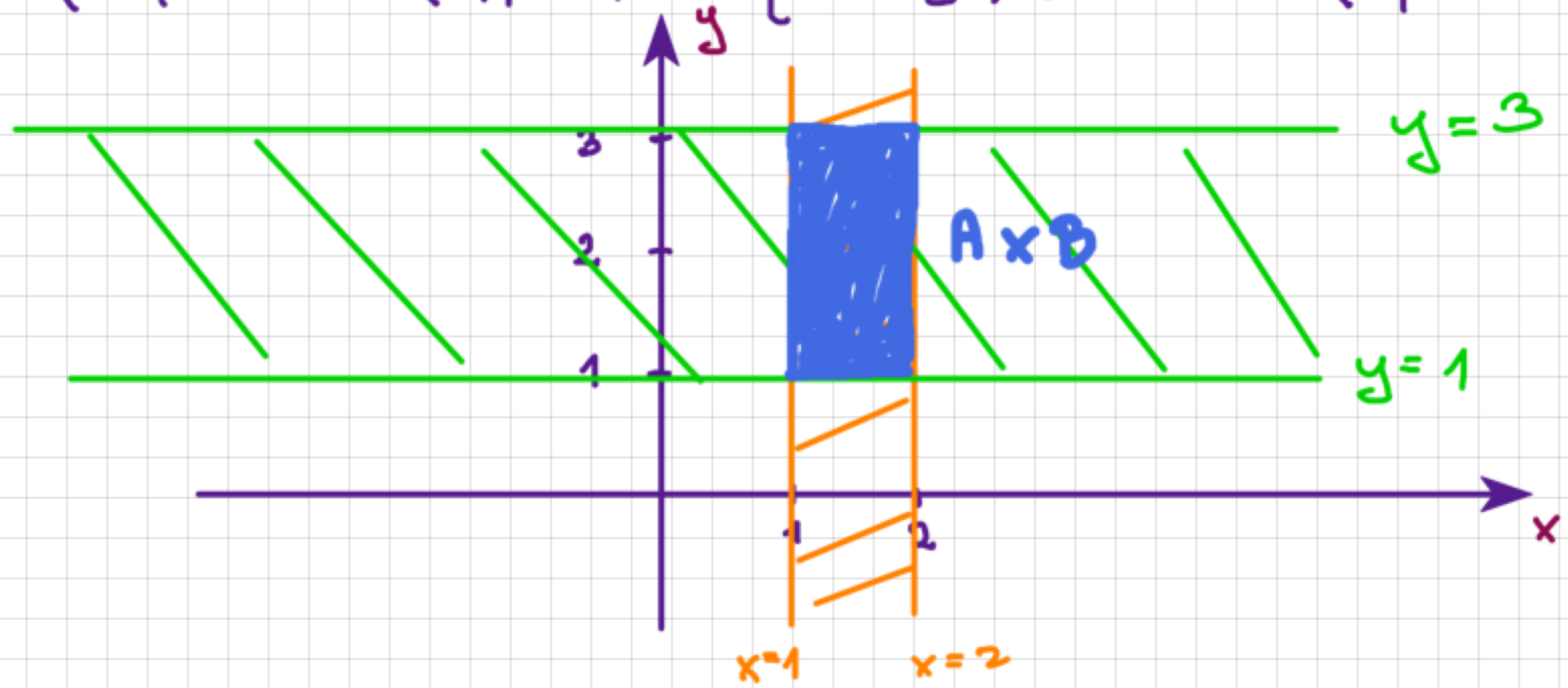
Jest to zbiór uporządkowanych par punktów (x,y) takich, że $x \in A$ i $x \in B$.

$$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{ (x,y) : \ x \in \mathbb{R} \ \wedge \ y \in \mathbb{R} \}$$

<u>Przykład</u> Na płaszczyźnie zaznaczymy zbiór $A \times B$, gdy

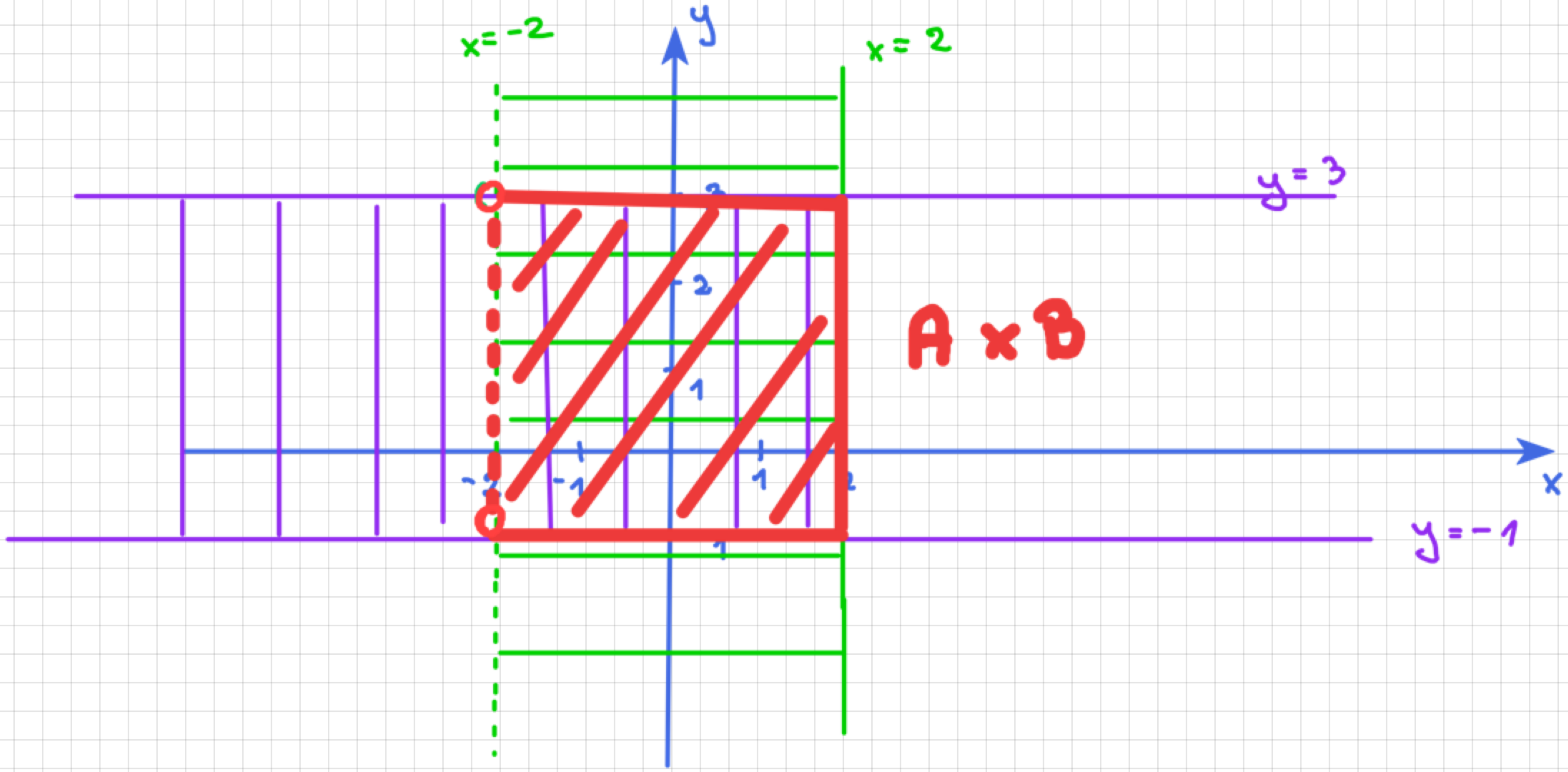
a) $A = \langle 1, 2 \rangle$, $\quad B = \langle 1, 3 \rangle$.

$$A \times B = \langle 1, 2 \rangle \times \langle 1, 3 \rangle = \{ (x,y) : \ x \in \langle 1, 2 \rangle \ \wedge \ y \in \langle 1, 3 \rangle \}$$



b) $A = (-2, 2\rangle$ $B = \langle -1, 3\rangle$

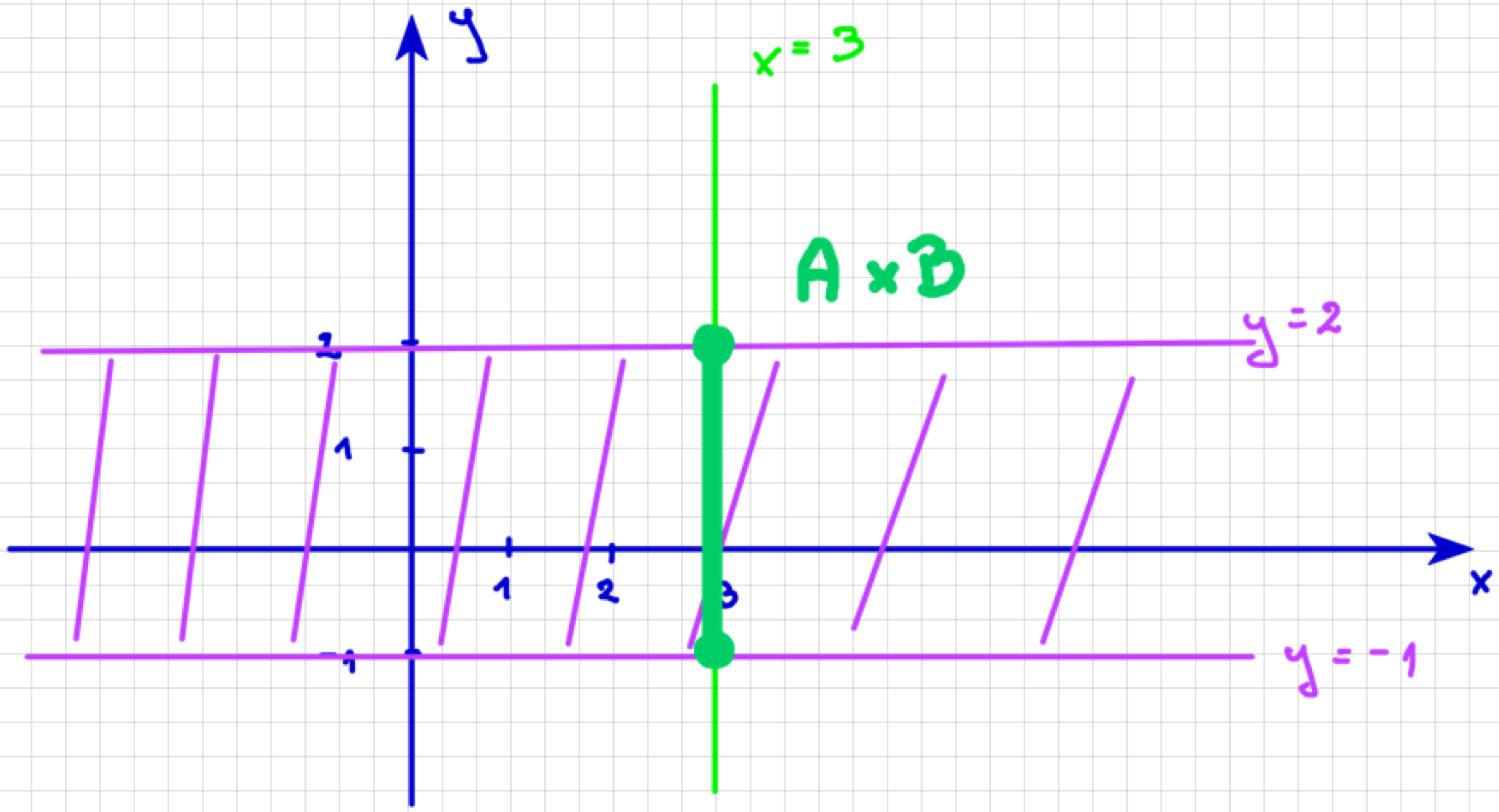
$$A \times B = (-2, 2\rangle \times \langle -1, 3\rangle = \{ (x, y) \in \mathbb{R}^2 : x \in (-2, 2\rangle \wedge y \in \langle -1, 3\rangle \}$$



c) $A = \{3\}$ $\quad B = \langle -1, 2 \rangle$

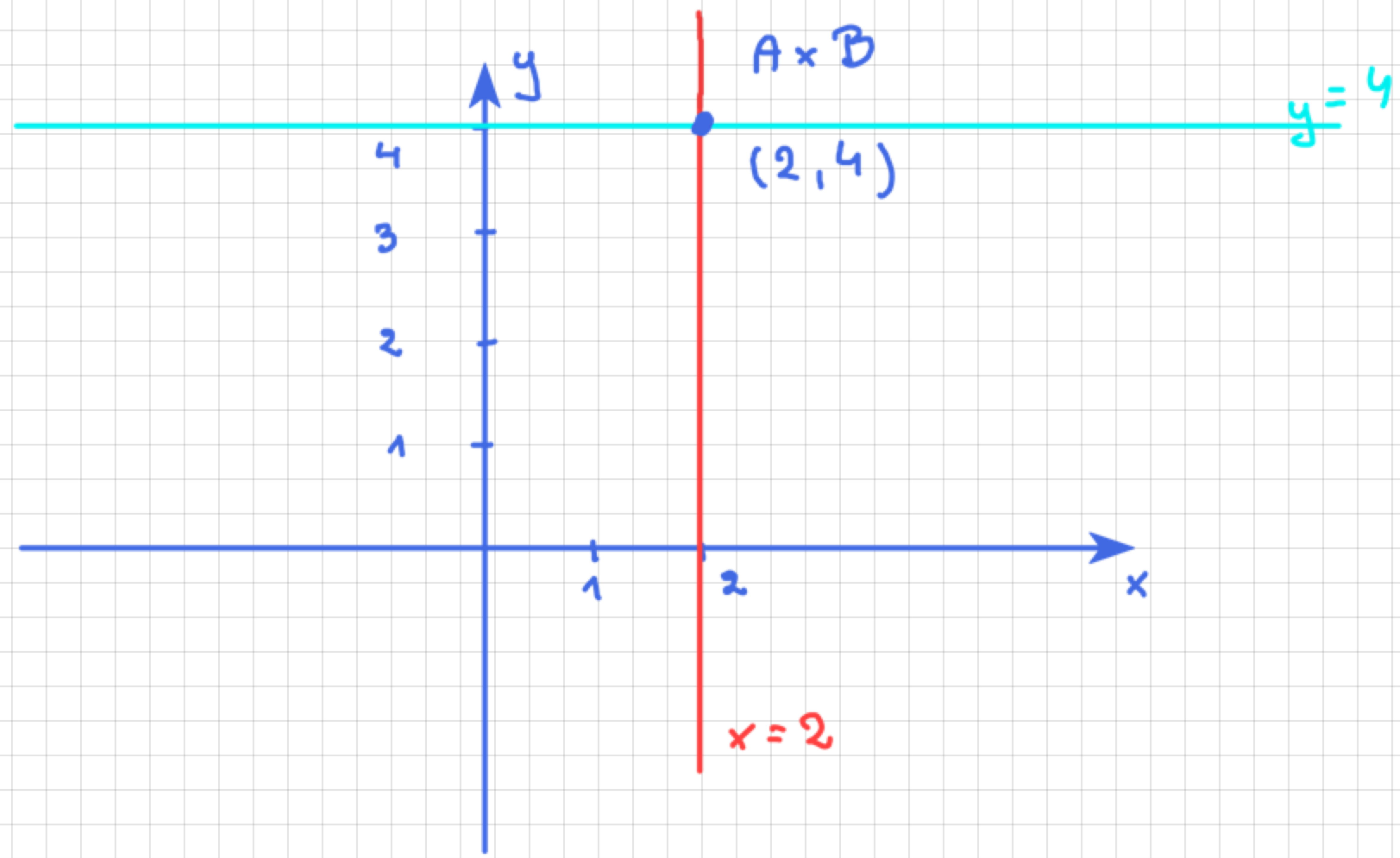
$$A \times B = \{3\} \times \langle -1, 2 \rangle = \{(x, y) \in \mathbb{R}^2 : x \in \{3\} \wedge y \in \langle -1, 2 \rangle\} =$$

$$= \{(x, y) \in \mathbb{R}^2 : x = 3 \wedge y \in \langle -1, 2 \rangle\}$$



d) $A = \{2\}$ $\quad B = \{4\}$

$A \times B = \{2\} \times \{4\} = \{(x,y) \in \mathbb{R}^2 : x = 2 \wedge y = 4\} = (2,4) \leftarrow$ punkt



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Przypomnienie

- Równanie okręgu o środku w $S=(x_0, y_0)$ i promieniu $r>0$:

$$(x-x_0)^2+(y-y_0)^2=r^2$$

np. $(x-1)^2+(y+3)^2=r^2$ – takie równanie opisuje okrąg o środku w $S=(1,-3)$ i promieniu $r>0$.

- Nierówność opisująca koło o środku w $S=(x_0, y_0)$ i promieniu $r>0$:

a) koło bez brzegu: $(x-x_0)^2+(y-y_0)^2<r^2$

b) koło z brzegiem: $(x-x_0)^2+(y-y_0)^2\leq r^2$

np. $(x-5)^2+(y+3)^2\leq 16$ $\quad S=(5,-3)$, $r=4$ (koło wraz z brzegiem)

$(x+1)^2+(y-7)^2<2$ $\quad S=(-1,7)$, $r=\sqrt{2}$ (koło bez brzegu)

Przykłady Znajdź środek i promień okręgu:

a) $x^2 + y^2 - 2x + 4y - 1 = 0$

$$\underbrace{x^2 - 2x} + \underbrace{y^2 + 4y} - 1 = 0$$

$$\underbrace{(x - 1)^2}_{x^2 - 2x + 1} - 1 + \underbrace{(y + 2)^2}_{y^2 + 4y + 4} - 4 - 1 = 0$$

$$(x-1)^2 + (y+2)^2 = 6 \qquad S = (1, -2) \qquad r = \sqrt{6}$$

b) $x^2 + y^2 - 10x + 8y = 0$

$$\underbrace{x^2 - 10x} + \underbrace{y^2 + 8y} = 0$$

$y^2 + 8y + 16$

$x^2 - 10x + 25$

$$(x - 5)^2 - 25 + \overbrace{(y + 4)^2} - 16 = 0$$

$$(x-5)^2 + (y+4)^2 = 41$$

$S = (5, -4) \qquad r = \sqrt{41}$.

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