22.10.2025

Zadanie Rozwiąż nierówność

a) $\log_{\frac{1}{2}}(2-x) < 1$

$D:\ 2-x > 0$

$2 > x$

$x < 2$

$D = (-\infty, 2)$

$\log_{\frac{1}{2}}(2-x) < \log_{\frac{1}{2}}\frac{1}{2}$

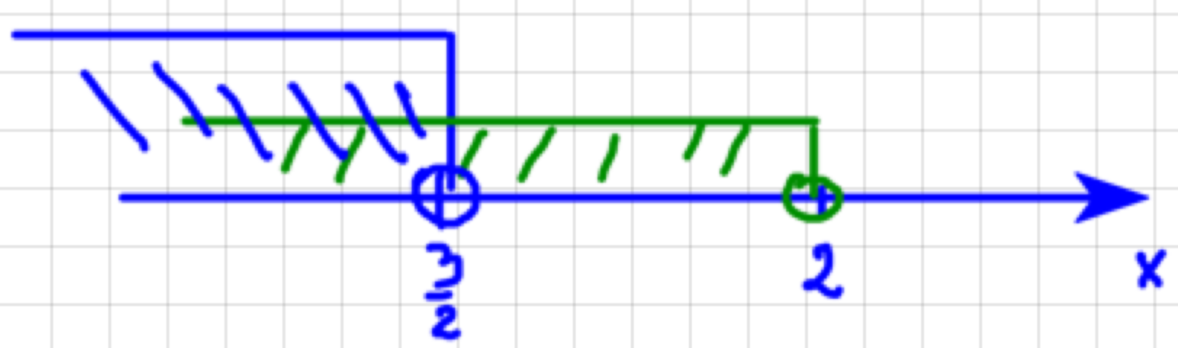
$2-x > \frac{1}{2}$

$-x > \frac{1}{2} - 2$

$-x > -\frac{3}{2} \quad | :(-1)$

$x < \frac{3}{2}$

Uwzględniając dziedzinę:



Odp.: $x \in (-\infty, \frac{3}{2})$

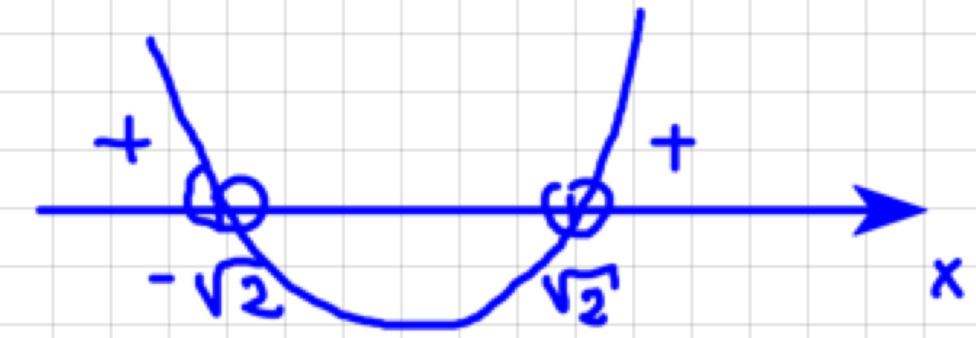
b) $\log_{\frac{1}{2}}(x^2-2) \geq -1$

$D: \quad x^2-2>0$

$x^2-2=0$

$x^2=2$

$x=\sqrt{2} \quad \vee \quad x=-\sqrt{2}$



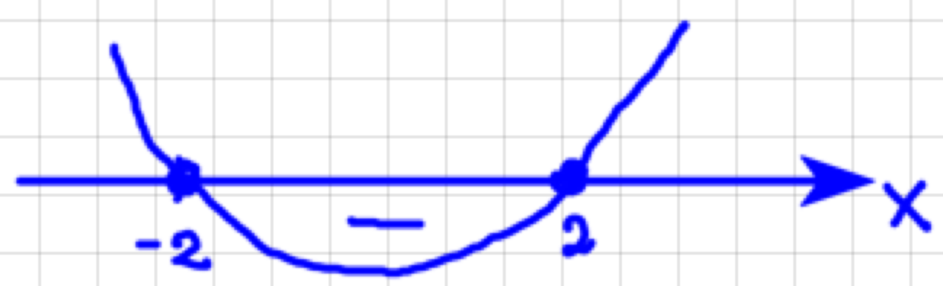
$D=(-\infty,-\sqrt{2})\cup(\sqrt{2},+\infty)$

$\log_{\frac{1}{2}}(x^2-2) \geq \log_{\frac{1}{2}} 2 \qquad \left(\frac{1}{2}\right)^{-1}$

$x^2-2 \leq 2$

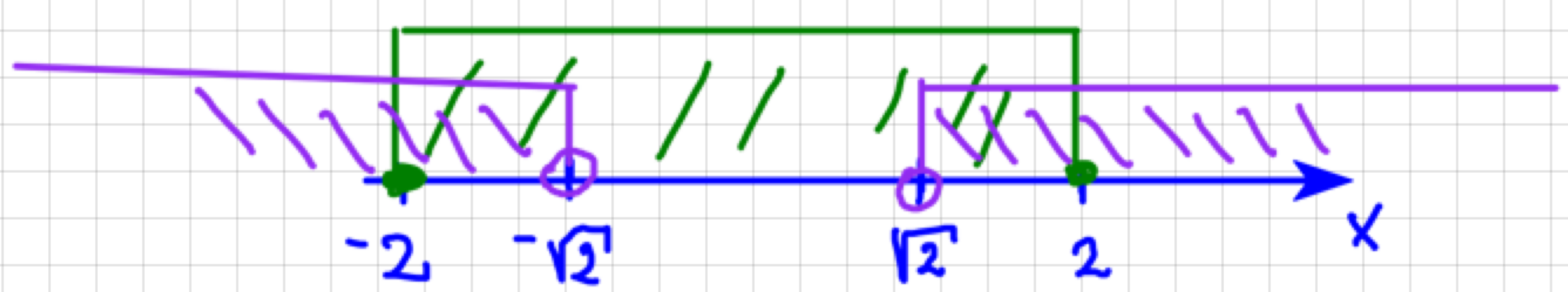
$x^2-4 \leq 0$

$x=2 \quad \vee \quad x=-2$



$x \in \langle -2, 2 \rangle$

Uwzględniamy dziedzinę:

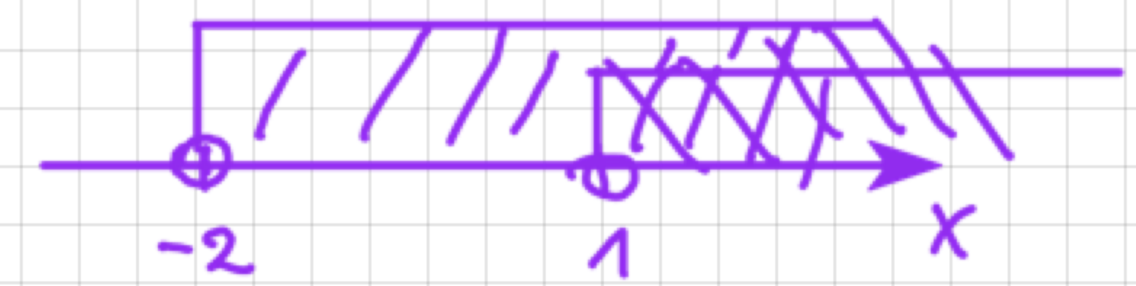


Odp.: $x \in \langle -2, -\sqrt{2}) \cup (\sqrt{2}, 2 \rangle$

c)

$$\log_4(x+2) - \log_4(x-1) \le 1$$

D: $\begin{cases} x+2>0 \\ x-1>0 \end{cases} \Leftrightarrow \begin{cases} x>-2 \\ x>1 \end{cases}$



$D = (1, +\infty)$

$$\log_4 \frac{x+2}{x-1} \le \log_4 4$$

$$\frac{x+2}{x-1} \le 4$$

$$\frac{x+2}{x-1} - 4 \le 0$$

$$\frac{x+2}{x-1} - \frac{4(x-1)}{(x-1)} \le 0$$

$$\frac{x+2-4x+4}{x-1} \le 0$$

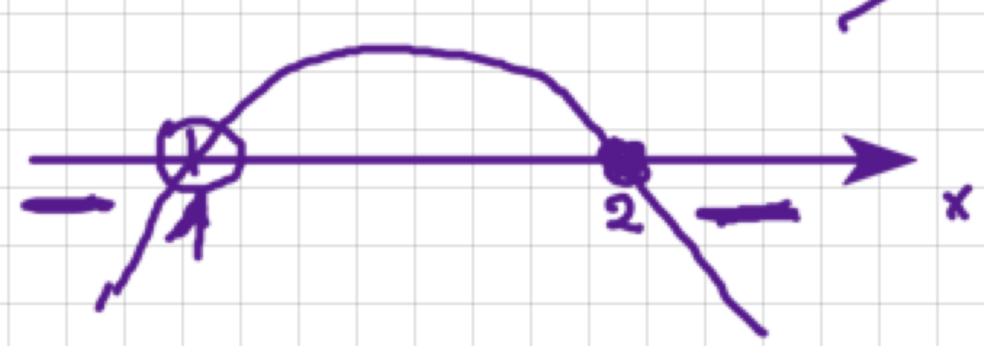
$$\frac{6-3x}{x-1} \le 0 \quad \Big| \cdot (x-1)^2$$

$$(6-3x)\cdot(x-1) \le 0$$

$6-3x=0 \quad \vee \quad x=1$

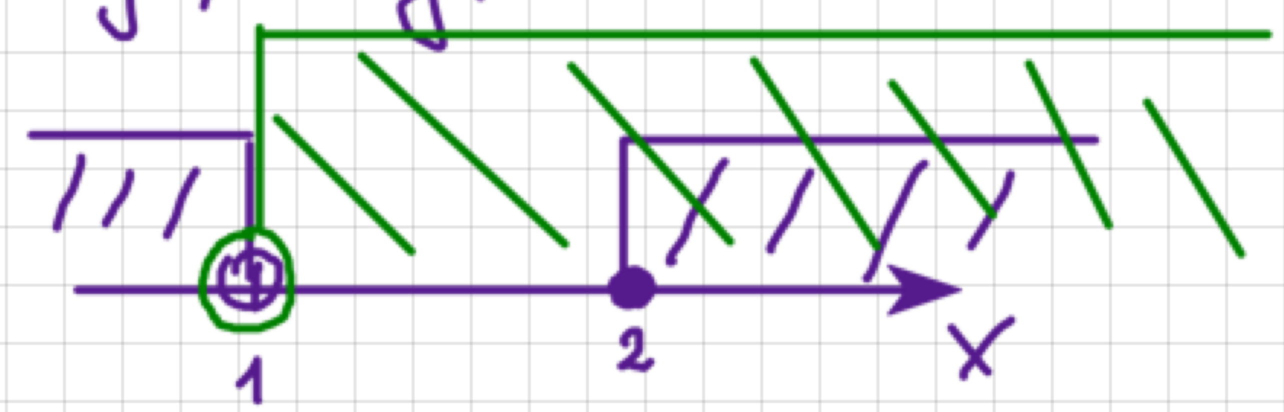
$6=3x$

$x=2$



$x \in (-\infty, +1) \cup \langle 2, +\infty)$

Uwzględniając dziedzinę:



Odp.: $x \in \langle 2, +\infty)$

d) $\log x + \log(x-1) > 0$

$$D: \begin{cases} x > 0 \\ x-1 > 0 \end{cases} \iff \begin{cases} x > 0 \\ x > 1 \end{cases} \iff x \in (1, +\infty)$$

$$D = (1, +\infty)$$

$$\log[x\cdot(x-1)] > 0$$

$$\log[x(x-1)] > \log_{10}(10^0)$$

$$\log[x(x-1)] > \log 1$$

$$x(x-1) > 1$$

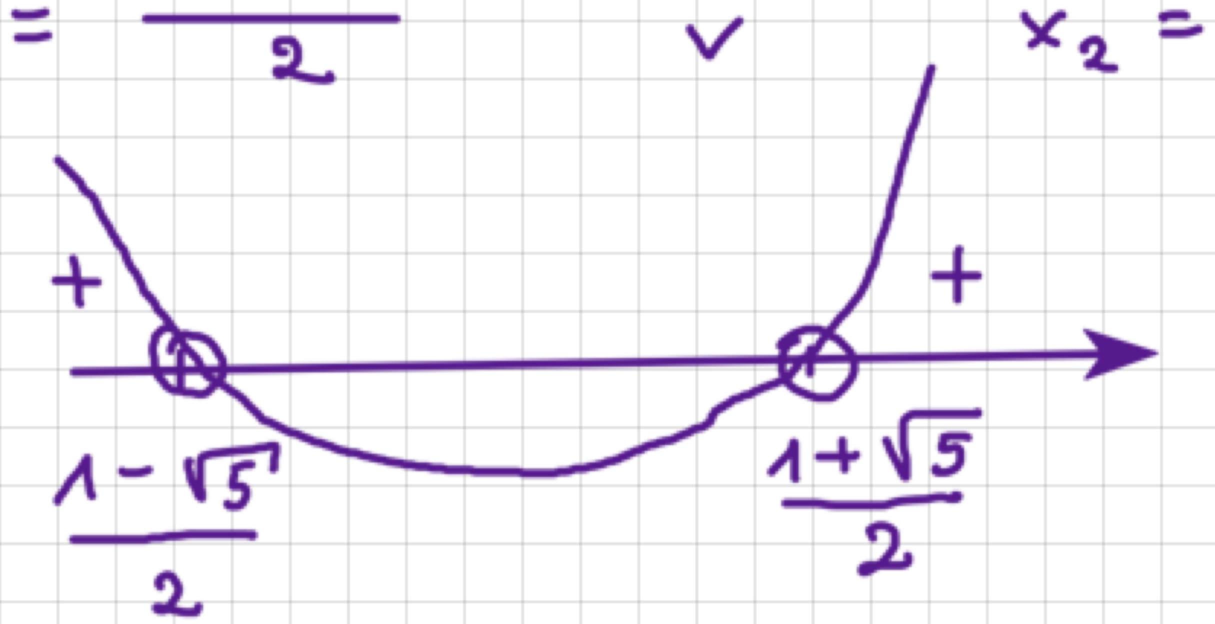
$$x^2 - x - 1 > 0$$

$$\Delta = 1 + 4 = 5, \quad \sqrt{\Delta} = \sqrt{5}$$

$x_1 \approx -0{,}6$

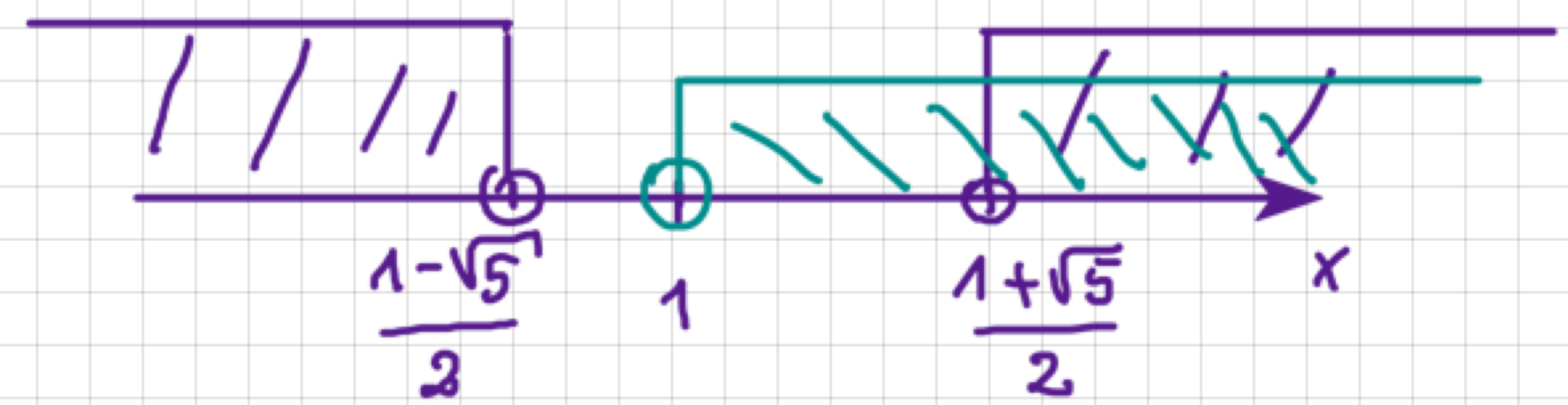
$$x_1 = \frac{1-\sqrt{5}}{2} \quad \vee \quad x_2 = \frac{1+\sqrt{5}}{2}$$

$x_2 \approx 1{,}6$



$$x \in \left(-\infty, \frac{1-\sqrt{5}}{2}\right) \cup \left(\frac{1+\sqrt{5}}{2}, +\infty\right)$$

Uwzględniamy dziedzinę



Odp.: $x \in \left(\frac{1+\sqrt{5}}{2}, +\infty\right)$

Funkcje trygonometryczne

$\pi = 180°$

	$0° = 0$	$30° = \frac{\pi}{6}$	$45° = \frac{\pi}{4}$	$60° = \frac{\pi}{3}$	$90° = \frac{\pi}{2}$	$180° = \pi$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1
$tg\, x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	X	0
$ctg\, x$	X	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	X

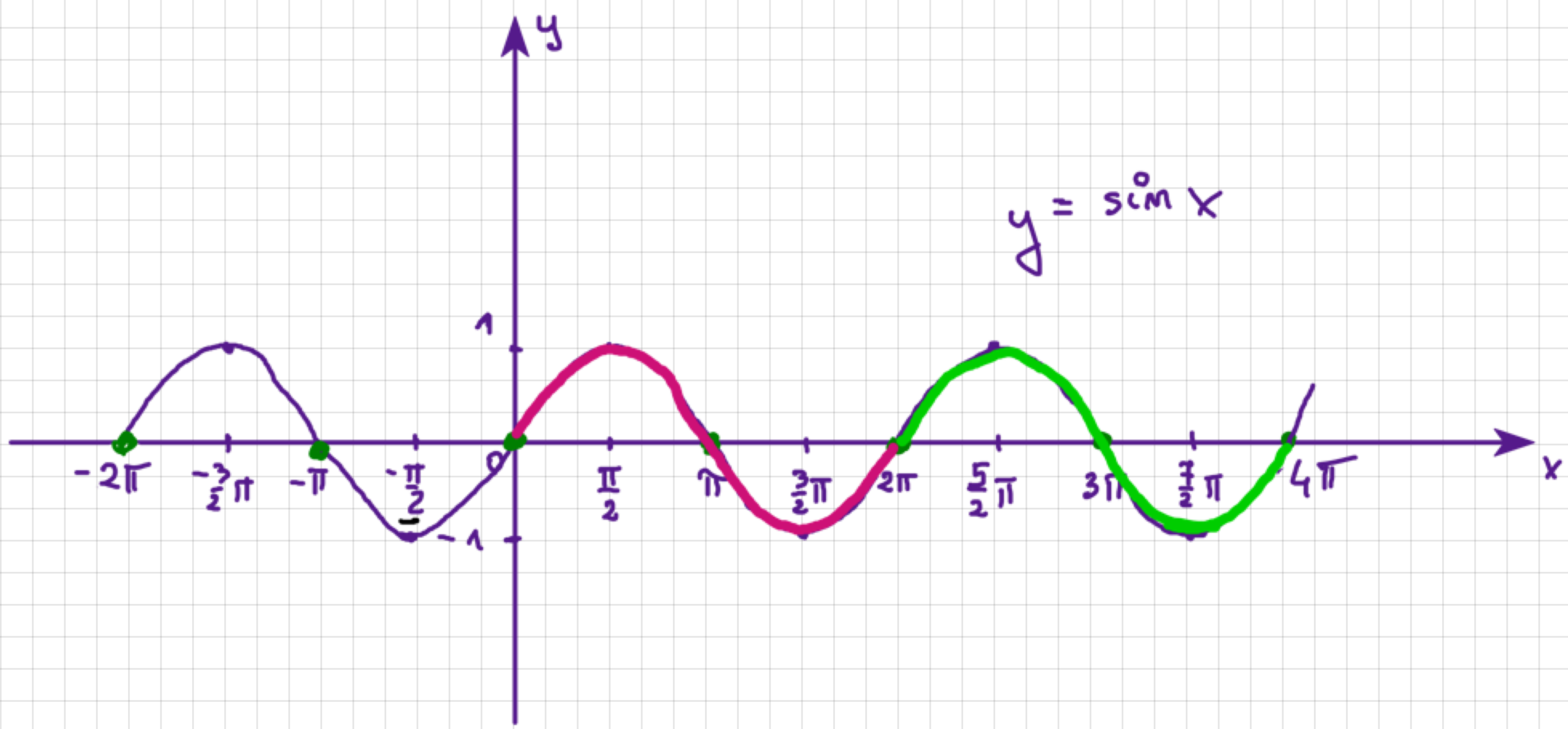
$$tg\, x = \frac{\sin x}{\cos x}$$

$$ctg\, x = \frac{\cos x}{\sin x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$tg\, x \cdot ctg\, x = 1$$

(1) $f(x) = \sin x \qquad x \in \mathbb{R} \quad , \quad y \in \langle -1, 1 \rangle$



Własności funkcji $f(x) = \sin x$

1) Miejsca zerowe: $x_0 = 0 + k \cdot \pi \quad , \quad k \in \mathbb{Z}$ (liczby całkowite)

2) Funkcja $\sin x$ jest funkcją nieparzystą, tzn. $\sin(-x) = -\sin x$

3) Jest to funkcja okresowa, o okresie podstawowym równym 2π.

Okres funkcji $\sin x$ wynosi $T = 2k\pi$, $k \in \mathbb{Z}$.