(1) $(x^4+\cos x)' = (x^4)' + (\cos x)' = 4x^3 + (-\sin x)$

$(f \pm g)' = f' \pm g'$

(2) $\left[\left(\frac{1}{x^2} - 3x\right)\cdot \operatorname{tg} x\right]' = \left(\frac{1}{x^2}-3x\right)'\cdot \operatorname{tg} x + \left(\frac{1}{x^2}-3x\right)(\operatorname{tg} x)' =$

$(f\cdot g)' = f'\cdot g + f\cdot g'$

$(cf)' = \underbrace{(c)'}_{=0}\cdot f + c\cdot f' = c\cdot f'$

$= \left(-\frac{2}{x^3} - 3\right)\cdot \operatorname{tg} x + \left(\frac{1}{x^2} - 3x\right)\cdot \frac{1}{\cos^2 x}$

$\left(\frac{1}{x^2}\right)' = (x^{-2})' = -2\cdot x^{-2-1} = -2x^{-3} = \frac{-2}{x^3}$

(3) $\left(\frac{x^2 - \arcsin x}{\sin x + \ln x}\right)' = \frac{\left(2x - \frac{1}{\sqrt{1-x^2}}\right)\cdot(\sin x + \ln x) - (x^2 - \arcsin x)\cdot\left(\cos x + \frac{1}{x}\right)}{(\sin x + \ln x)^2}$

$\left(\frac{f}{g}\right)' = \frac{f'\cdot g - f\cdot g'}{g^2}$

(1) $(x^\alpha)' = \alpha \cdot x^{\alpha - 1}$

$(\sqrt{x})' = (x^{\frac{1}{2}})' = \frac{1}{2}\cdot x^{-\frac{1}{2}} = \frac{1}{2}\cdot\frac{1}{\sqrt{x}}$

$\left(\frac{1}{x}\right)' =$

(2) $(\cos x)' = -\sin x$

(3) $(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$\cos^2 x = (\cos x)^2$

$\cos x^2 = \cos(x^2)$

(4) $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$

(5) $(\sin x)' = \cos x$

(6) $(\ln x)' = \frac{1}{x}$

(4) $\left(\sqrt{x+e^x}\right)' = \frac{1}{2\sqrt{x+e^x}}\cdot(1+e^x)$

f-cja zewnętrzna $\rightarrow z = \sqrt{w}$ $\qquad z' = \frac{1}{2\sqrt{w}}$

$\rightarrow w = x + e^x$ $\qquad w' = 1 + e^x$

f-cja wewnętrzna

(5) $\left(\frac{3}{\ln^2 x} + \operatorname{arctg} 5x\right)' = \frac{-6}{(\ln x)^3}\cdot\frac{1}{x} + \frac{5}{1+25x^2}$

$\left(\frac{3}{\ln^2 x}\right)' = \left(3\,\underbrace{(\ln x)^{-2}}\right)' = 3\cdot\frac{(-2)}{(\ln x)^3}\cdot\frac{1}{x}$

$z = w^{-2}$ $\qquad z' = (-2)w^{-3} = \frac{-2}{w^3}$

$w = \ln x$ $\qquad w' = \frac{1}{x}$

$(\operatorname{arctg} 5x)' = \frac{1}{1+(5x)^2}\cdot 5$

$z = \operatorname{arctg} w$ $\qquad z' = \frac{1}{1+w^2}$

$w = 5x$ $\qquad w' = 5$

(7) $(e^x)' = e^x$

(8) $(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

(9) $(a^x)' = a^x \cdot \ln a$

(9') $(e^x)' = e^x \cdot \underbrace{\ln e}_{=1} = e^x$

$\ln^2 x = (\ln x)^2$

$\ln x^2 = \ln(x^2)$

(6) $\left[\left(2^x - \ln(\sin x)\right)\cdot \cos^3 x\right]' =$

$= \left(2^x \ln a - \frac{1}{\sin x}\cdot\cos x\right)\cdot\cos^3 x + \left(2^x - \ln(\sin x)\right)\cdot 3(\cos x)^2\cdot(-\sin x)$

$(a^x)' = a^x\cdot\ln a$

$\left[(\cos x)^3\right]' = ?$

$z = \ln w$ $\qquad z' = \frac{1}{w}$

$w = \sin x$ $\qquad w' = \cos x$

$z = w^3$ $\qquad z' = 3w^2$

$w = \cos x$ $\qquad w' = -\sin x$

(7) $\left(\dfrac{\log_4 x + \sqrt{x} + 3}{1 + e^{2x-1}}\right)' = \dfrac{\left(\frac{1}{x\ln 4} + \frac{1}{2\sqrt{x}}\right)(1+e^{2x-1}) - (\log_4 x + \sqrt{x} + 3)\cdot e^{2x-1}\cdot 2}{(1+e^{2x-1})^2}$

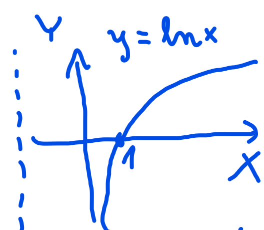
$z = e^w \qquad z' = e^w$
$w = 2x-1 \qquad w' = 2$

$(\log_a x)' = \dfrac{1}{x\ln a}$
$(\ln x)' = (\log_e x)' =$
$= \dfrac{1}{x\underbrace{\ln e}_{=1}} = \dfrac{1}{x}$

Tw. de l'Hospitala

$\dfrac{0}{0}$, $\dfrac{\infty}{\infty}$

(1) $\lim\limits_{x\to 1} \dfrac{x^2-1}{\ln x} = \overset{S.N}{\left\{\dfrac{0}{0}\right\}} \overset{H}{=} \lim\limits_{x\to 1} \dfrac{(x^2-1)'}{(\ln x)'} = \lim\limits_{x\to 1} \dfrac{2x}{\frac{1}{x}} =$
$= \left\{\dfrac{2}{1}\right\} = 2$



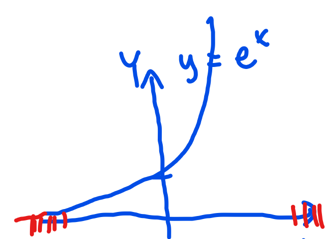
$\ln 1 = 0$

(2) $\lim\limits_{x\to 0^-} \dfrac{\operatorname{arctg}(2x)}{x^2-x} = \overset{S.N.}{\left\{\dfrac{0}{0}\right\}} \overset{H}{=} \lim\limits_{x\to 0^-} \dfrac{(\operatorname{arctg}(2x))'}{(x^2-x)'} =$

$\operatorname{arctg} 0 = 0$

$= \lim\limits_{x\to 0^-} \dfrac{\frac{1}{1+4x^2}\cdot 2}{2x-1} = \dfrac{2}{-1} = -2$

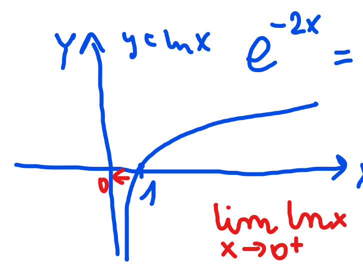
(3) $\lim\limits_{x\to +\infty} (x+3)e^{-2x} = \{(+\infty)\cdot e^{-\infty}\} = \overset{S.N}{\{(+\infty)\cdot 0\}} =$
$= \lim\limits_{x\to +\infty} \dfrac{x+3}{e^{2x}} = \overset{S.N}{\left\{\dfrac{+\infty}{+\infty}\right\}} \overset{H}{=} \lim\limits_{x\to +\infty} \dfrac{(x+3)'}{(e^{2x})'} =$
$= \lim\limits_{x\to +\infty} \dfrac{1}{e^{2x}\cdot 2} = \left\{\dfrac{1}{+\infty}\right\} = 0$



$\lim\limits_{x\to -\infty} e^x = 0$

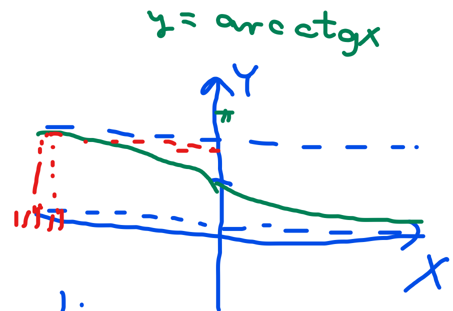
$e^{-2x} = \dfrac{1}{e^{2x}}$

(4) $\lim\limits_{x\to 0^+} x^2\cdot \ln x = \overset{S.N.}{\{0\cdot(-\infty)\}} = \lim\limits_{x\to 0^+} \dfrac{\ln x}{\frac{1}{x^2}} =$
$= \overset{S.N}{\left\{\dfrac{-\infty}{+\infty}\right\}} \overset{H}{=} \lim\limits_{x\to 0^+} \dfrac{(\ln x)'}{(x^{-2})'} = \lim\limits_{x\to 0^+} \dfrac{\frac{1}{x}}{-2x^{-3}} = \lim\limits_{x\to 0^+} \dfrac{\frac{1}{x}}{\frac{-2}{x^3}}$



$\lim\limits_{x\to 0^+} \ln x = -\infty$

$$= \lim_{x\to 0^+} \frac{1}{x}\cdot\frac{x^3}{(-2)} = \lim_{x\to 0^+}\left(-\frac{1}{2}\right)x^2 = 0$$



$$\lim_{x\to -\infty} \operatorname{arcctg} x = \pi$$

(5) $$\lim_{x\to -\infty} x\left(\operatorname{arcctg} x - \pi\right) = \{(-\infty)\cdot 0\} =$$

$$= \lim_{x\to -\infty} \frac{\operatorname{arcctg} x - \pi}{\frac{1}{x}} = \overset{S.N}{\left\{\frac{0}{0}\right\}} \overset{H}{=} \ldots\ldots$$

(6) $$\lim_{x\to +\infty}\left(\sqrt{3x+1} - e^{2x}\right) = \overset{S.N.}{\{\infty - \infty\}} = \lim_{x\to +\infty} \sqrt{3x+1}\left(1 - \frac{e^{2x}}{\sqrt{3x+1}}\right) = \ldots$$

$$= \ldots\ldots$$

(7) $$\lim_{x\to 0^+} x^2\cdot e^{\frac{1}{x}} = \overset{S.N}{\{0\cdot\infty\}} = \lim_{x\to 0^+} \frac{e^{\frac{1}{x}}}{\frac{1}{x^2}} = \overset{S.N}{\left\{\frac{\infty}{\infty}\right\}} = \ldots$$