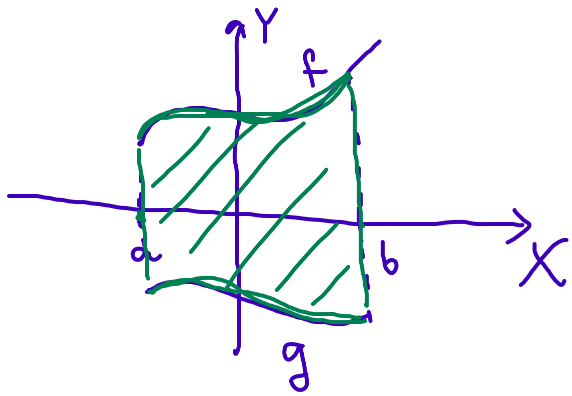
Tw. Newtona-Leibniza. Jeżeli $f:[a,b]\to\mathbb{R}$ jest funkcją ciągłą i F jest dowolną funkcją pierwotną funkcji f, to

$$\int_a^b f(x)\,dx = F(b)-F(a) \qquad \Big\{\; F(b)-F(a) = \big[F(x)\big]\Big|_a^b$$

$$\int_{-1}^{\sqrt{3}} \frac{1}{1+x^2}\,dx = \big[\operatorname{arctg} x\big]\Big|_{-1}^{\sqrt{3}} = \operatorname{arctg}(\sqrt{3}) - \operatorname{arctg}(-1) = \frac{\pi}{3} - \left(-\frac{\pi}{4}\right) = \frac{7}{12}\pi$$

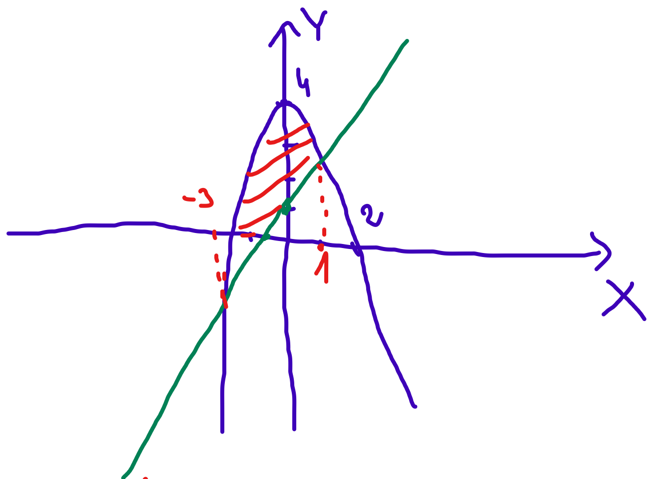
$$\int \frac{1}{1+x^2}\,dx = \operatorname{arctg} x + C$$



$$P = \int_a^b \big(f(x)-g(x)\big)\,dx$$

OBLICZYĆ POLE OBSZARU OGRANICZONEGO KRZYWYMI

(1) $y = 4-x^2$, $y = 2x+1$



$$\begin{cases} y = 4-x^2 \\ y = 2x+1 \end{cases}$$

$4-x^2 = 2x+1$

$-x^2-2x+3=0$

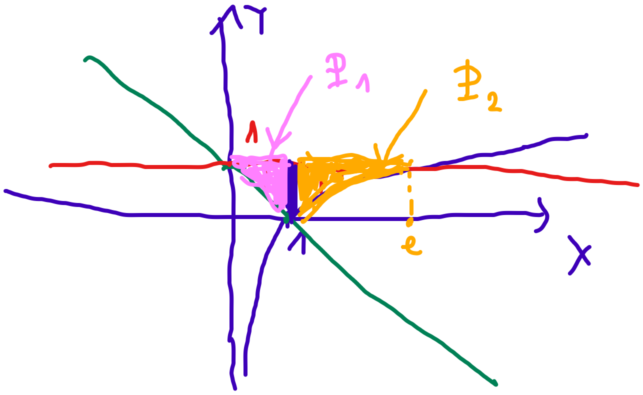
$\Delta = 4-4\cdot(-1)\cdot 3 = 16$

$x_1 = \frac{2+4}{-2} = -3$, $x_2 = \frac{2-4}{-2} = 1$

$$P = \int_{-3}^{1} \big(4-x^2-(2x+1)\big)\,dx = \int_{-3}^{1}\left(3-2x-x^2\right)dx = \left[3x - x^2 - \frac{x^3}{3}\right]\Big|_{-3}^{1} =$$

$$= 3-1-\frac{1}{3} - \left(-9-\cancel{9}+\cancel{9}\right) = 2-\frac{1}{3}+9 = \frac{32}{3}$$

(2) $y = \ln x \ , \ y = 1 - x \ , \ y = 1$



$P = P_1 + P_2$

$P_1 = P_\Delta = \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$

$$P_1 = \int_0^1 (1 - (1 - x))\, dx = \int_0^1 x\, dx = \left[\frac{x^2}{2} \right]\Big|_0^1 = \frac{1^2}{2} - \frac{0^2}{2} = \frac{1}{2}$$

$$\begin{cases} y = 1 \\ y = \ln x \end{cases}$$

$1 = \ln x$

$x = e^1 = e$

$$P_2 = \int_1^e (1 - \ln x)\, dx = \left[x - (x \ln x - x) \right]\Big|_1^e = \left[2x - x \ln x \right]\Big|_1^e =$$

$$= 2e - e \underbrace{\ln e}_{=1} - 2 + 1 \cdot \underbrace{\ln 1}_{=0} = e - 2$$

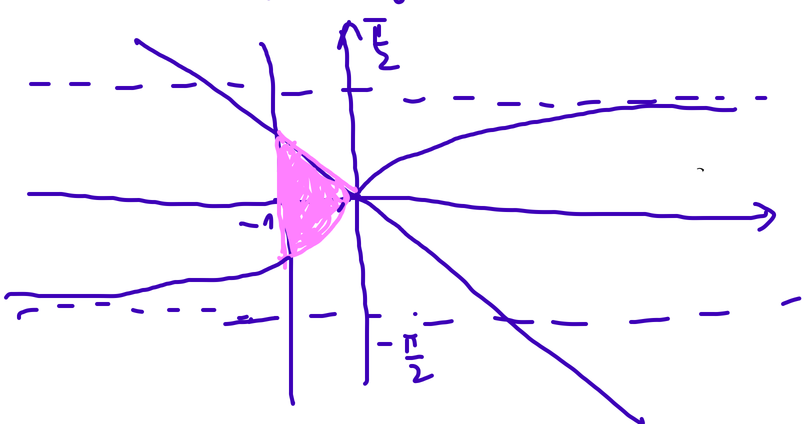
$$\int 1 \cdot \ln x\, dx = \left\{ \begin{matrix} f(x) = \ln x & g'(x) = 1 \\ f'(x) = \frac{1}{x} & g(x) = x \end{matrix} \right\} = x \ln x - \int \frac{1}{x} \cdot x\, dx = x \ln x - \int 1 dx =$$

$$= x \ln x - x + C$$

$$\int f \cdot g' = f \cdot g - \int f' \cdot g$$

$$P = P_1 + P_2 = \frac{1}{2} + e - 2 = e - \frac{3}{2}$$

(3) $y = \operatorname{arctg} x \ , \ y = -x \ , \ x = -1$



$$P = \int_{-1}^{0} (-x - \operatorname{arctg} x)\,dx = \left[-\frac{x^2}{2} - x \operatorname{arctg} x + \frac{1}{2}\ln(1+x^2) \right] \Big|_{-1}^{0} =$$

$$= 0 - \left(-\frac{1}{2} + \operatorname{arctg}(-1) + \frac{1}{2}\ln 2 \right) = \frac{1}{2} - \left(-\frac{\pi}{4}\right) - \frac{1}{2}\ln 2 = \frac{1}{2} + \frac{\pi}{4} - \frac{1}{2}\ln 2$$

$$\int 1 \cdot \operatorname{arctg} x\,dx = \left\{ \begin{array}{ll} f(x) = \operatorname{arctg} x & g'(x) = 1 \\ f'(x) = \frac{1}{1+x^2} & g(x) = x \end{array} \right\} = x \operatorname{arctg} x - \int \frac{1}{1+x^2} \cdot x\,dx$$

$$= x \operatorname{arctg} x - \frac{1}{2} \int \frac{2x}{1+x^2}\,dx = x \operatorname{arctg} x - \frac{1}{2}\ln|1+x^2| + C =$$

$$\left\{ \int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C \right.$$

$$\bigwedge_{x \in \mathbb{R}} 1+x^2 > 0$$

$$= x \operatorname{arctg} x - \frac{1}{2}\ln(1+x^2) + C$$