Granice ciągów

1. $\lim\limits_{n\to\infty} \frac{3^{n+1} - 4^n + 2\cdot 8^{n+1}}{2^{2n+3} - 9^{n+1} + 1} = \left\{ \frac{\infty - \infty + \infty}{\infty - \infty + 1} \right\} =$

$a^n \xrightarrow[n\to\infty]{} +\infty$

$a>1$ $\quad a^n \xrightarrow[n\to\infty]{} 0$

$a\in(0,1)$

$(a^b)^c = a^{bc}$

$\frac{a^b}{c^b} = \left(\frac{a}{c}\right)^b$

$= \lim\limits_{n\to\infty} \frac{3^n\cdot 3^1 - 4^n + 2\cdot 8^n\cdot 8^1}{2^{2n}\cdot 2^3 - 9^n\cdot 9 + 1} =$

$= \lim\limits_{n\to\infty} \frac{3^n - 4^n + 16\cdot 8^n}{4^n\cdot 8 - 9\cdot 9^n + 1} =$

SN $\left\{\frac{16\cdot\infty}{\infty}\right\}$

$\cancel{\frac{4^n}{9^n}}$ $\cancel{\frac{8^n}{9^n}}$

$= \lim\limits_{n\to\infty} \frac{9^n\left(\frac{1}{3} - \frac{4^n}{9^n} + \frac{16\cdot 8^n}{9^n}\right)}{9^n\left(\frac{4^n\cdot 8}{9^n} - 9 + \frac{1}{9^n}\right)} =$

$= \lim\limits_{n\to\infty} \frac{\frac{1}{3} - \left(\frac{4}{9}\right)^n + 16\cdot\left(\frac{8}{9}\right)^n}{8\cdot\left(\frac{4}{9}\right)^n - 9 + \frac{1}{9^n}} = \frac{\frac{1}{3}\cdot 0 + 16\cdot 0}{8\cdot 0 - 9 + 0} = \frac{\frac{1}{3}}{-9} = -\frac{1}{27}$

$\sqrt[n]{n} \xrightarrow[n\to\infty]{} 1$

$a_n \xrightarrow[n\to\infty]{} a \in (0,\infty) \Rightarrow \sqrt[n]{a_n} \xrightarrow[n\to\infty]{} 1$

kilka dodatnich

2. $\lim\limits_{n\to\infty} \sqrt[n]{\frac{6n - 3n^2 + n^3}{2^{n+1} - 3}} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{n^3 - 3n^2 + 6n}{2^n\cdot 2 - 3}} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{n^3\left(1 - \frac{3}{n} + \frac{6}{n^2}\right)}{2^n\left(2 - \frac{3}{2^n}\right)}} = \lim\limits_{n\to\infty} \frac{\sqrt[n]{n^3}}{\sqrt[n]{2^n}}\cdot\sqrt[n]{\frac{1 - \frac{3}{n} + \frac{6}{n^2}}{2 - \frac{3}{2^n}}} = \lim\limits_{n\to\infty} \frac{\sqrt[n]{n}}{2}\cdot\sqrt[n]{\frac{1 - \frac{3}{n} + \frac{6}{n^2}}{2 - \frac{3}{2^n}}} = \frac{1^3}{2}\cdot 1 = \frac{1}{2}$

$\frac{1}{\frac{0}{a}} = \frac{b}{a}$

$\frac{1}{2} \in (0,\infty)$

3. $\lim\limits_{n\to\infty} \left(\frac{2n^2 + 3n - 1}{2n^2 + 4}\right)^{n-1} = \{1^\infty\} =$

$a_n \xrightarrow[n\to\infty]{} \neq \infty \Rightarrow \left(1 + \frac{1}{a_n}\right)^{a_n} \xrightarrow[n\to\infty]{} e$

$= \lim\limits_{n\to\infty} \left(\frac{2n^2 + 4 + 3n - 5}{2n^2 + 4}\right)^{n-1} = \lim\limits_{n\to\infty} \left(\frac{2n^2 + 4}{2n^2 + 4} + \frac{3n - 5}{2n^2 + 4}\right)^{n-1} = \lim\limits_{n\to\infty} \left(1 + \frac{1}{\frac{2n^2 + 4}{3n - 5}}\right)^{\frac{2n^2 + 4}{3n - 5}\cdot\frac{3n - 5}{2n^2 + 4}\cdot(n-1)} =$

$n-1$

$\frac{(3n - 5)(n - 1)}{2n^2 + 4} \to \frac{3}{2}$

$= \lim\limits_{n\to\infty} \left[\left(1 + \frac{1}{\frac{2n^2 + 4}{3n - 5}}\right)^{\frac{2n^2 + 4}{3n - 5}}\right]^{\frac{3}{2}} = e^{\frac{3}{2}} = \sqrt{e^3} = e\sqrt{e}$

$\frac{2n^2 + 4}{3n - 5} \to \infty$, $\to e$

$\lim\limits_{n\to\infty} \frac{2n^2 + 4}{3n - 5} = \lim\limits_{n\to\infty} \frac{n^2\left(2 + \frac{4}{n^2}\right)}{n\left(3 - \frac{5}{n}\right)} = \lim\limits_{n\to\infty} \frac{n\left(2 + \frac{4}{n^2}\right)}{3 - \frac{5}{n}} = \left\{\frac{\infty\,(2 + 0)}{3 - 0}\right\} = \infty$

$\lim\limits_{n\to\infty} \frac{(3n - 5)(n - 1)}{2n^2 + 4} = \lim\limits_{n\to\infty} \frac{n\left(3 - \frac{5}{n}\right)n\left(1 - \frac{1}{n}\right)}{n^2\left(2 + \frac{4}{n^2}\right)} = \left\{\frac{(3 - 0)(1 - 0)}{2 + 0}\right\} = \frac{3}{2}$

Asymptoty

1. $f(x) = x^2\ln x$ $\qquad D: x > 0$, $D = (0,\infty)$

$\left(\frac{1}{x^2}\right)' = \left(x^{-2}\right)' = -2x^{-3}$

f - ciągła

as. pionowe (?)

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} x^2\ln x = \left\{0^2\cdot\ln 0^+ = 0\cdot(-\infty)\right\} \overset{SN}{=} \lim\limits_{x\to 0^+} \frac{\ln x}{\frac{1}{x^2}} = \left\{\frac{-\infty}{\frac{1}{0^+}} = \frac{-\infty}{+\infty}\right\} \overset{H}{=} \lim\limits_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{2}{x^3}} = \lim\limits_{x\to 0^+}\left(-\frac{1}{x}\cdot\frac{x^3}{2}\right) = \lim\limits_{x\to 0^+}\left(-\frac{x^2}{2}\right) = 0$

Brak asymptot pionowych.

as. poziome (?)

$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} x^2\ln x = \{\infty\cdot\infty\} = \infty$

Brak asymptot poziomych.

as. ukośna (?)

$\lim\limits_{x\to+\infty} \frac{f(x)}{x} = \lim\limits_{x\to\infty} \frac{x^2\ln x}{x} = \lim\limits_{x\to\infty} x\ln x = \{+\infty\cdot\infty\} = +\infty$

Brak asymptot ukośnych.

2. $f(x) = e^{\frac{x+3}{x-2}}$ $\qquad D: x - 2 \neq 0$, $x \neq 2$, $D = \mathbb{R}\setminus\{2\}$

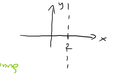
funkcja potęgowa

as. pionowe (?)

$\lim\limits_{x\to 2} e^{\frac{x+3}{x-2}} = \left\{e^{\frac{5}{0}}\right\}$ nie istnieje

$\lim\limits_{x\to 2^-} e^{\frac{x+3}{x-2}} = \left\{e^{\frac{5}{0^-}} = e^{-\infty}\right\} = 0$

$\lim\limits_{x\to 2^+} e^{\frac{x+3}{x-2}} = \left\{e^{\frac{5}{0^+}} = e^{+\infty}\right\} = +\infty \Rightarrow$ Prosta $x = 2$ jest asymptotą pionową prawostronną wykresu funkcji f.



as. poziome (?)

$\lim\limits_{x\to+\infty} e^{\frac{x+3}{x-2}} = \left\{e^{\frac{\infty+3}{\infty-2}} = e^{\frac{\infty}{\infty}}\right\} \overset{SN}{=} \lim\limits_{x\to\infty} e^{\frac{x\left(1+\frac{3}{x}\right)}{x\left(1-\frac{2}{x}\right)}} = \lim\limits_{x\to\infty} e^{\frac{1+\frac{3}{x}}{1-\frac{2}{x}}} = \left\{e^{\frac{1+0}{1-0}} = e^1\right\} = e$

$\lim\limits_{x\to-\infty} e^{\frac{x+3}{x-2}} = \lim\limits_{x\to-\infty} e^{\frac{1+\frac{3}{x}}{1-\frac{2}{x}}} = \left\{e^{\frac{1+0}{1-0}}\right\} = e$

$\lim/\lim = \infty$ $\quad y = e$

$y = e$

$\lim/\lim = \infty$

Prosta $y = e$ jest asymptotą poziomą wykresu funkcji f.

poziomą

Granice funkcji:

1. $\lim\limits_{x\to-\infty} \left(x + \arcsin\frac{x}{2}\right) = \left\{-\infty + \arcsin 0 = -\infty + 0\right\} = -\infty$

$(a-b)(a+b) = a^2 - b^2$

2. $\lim\limits_{x\to\infty} \ln\left(2x - \sqrt{x^2 - 1}\right) = \left\{\ln\left(\infty - \sqrt{\infty - 1}\right) = \ln(\infty - \infty)\right\} \overset{SN}{=} \lim\limits_{x\to\infty} \ln\left(\frac{\left(2x - \sqrt{x^2-1}\right)\cdot\frac{2x+\sqrt{x^2-1}}{2x+\sqrt{x^2-1}}}{2x+\sqrt{x^2-1}}\right) = \lim\limits_{x\to\infty} \ln\frac{(2x)^2 - \left(\sqrt{x^2-1}\right)^2}{2x+\sqrt{x^2-1}} = \lim\limits_{x\to\infty} \ln\frac{4x^2 - (x^2-1)}{2x+\sqrt{x^2-1}} = \lim\limits_{x\to\infty} \ln\frac{3x^2+1}{2x+\sqrt{x^2-1}} = \left\{\ln\frac{\infty}{\infty+\infty}\right\} = \ln\frac{\infty}{3} = \left\{\lim\limits_{x\to\infty} \ln\frac{x^2\left(3+\frac{1}{x^2}\right)}{x\left(2+\sqrt{1-\frac{1}{x^2}}\right)}\right\} = \left\{\ln\frac{\infty(3+0)}{2+\sqrt{1-0}}\right\} = \ln\frac{\infty}{3} = \ln\infty\right\} = +\infty$

$\sqrt{x^2-1} = \sqrt{x^2\left(1-\frac{1}{x^2}\right)} = x\sqrt{1-\frac{1}{x^2}}$

3. $\lim\limits_{x\to e} \frac{\cos x}{1 - \ln x} = \left\{\frac{\cos e}{1 - \ln e}\right\} = \frac{\cos e}{0}$ nie istnieje

$\lim\limits_{x\to e^-} \frac{\cos x}{1 - \ln x} = \left\{\frac{\cos e}{1 - 1^-} = \frac{\cos e}{0^+}\right\} = -\infty$

$\cos e < 0$

$\lim\limits_{x\to e^+} \frac{\cos x}{1 - \ln x} = \left\{\frac{\cos e}{1 - 1^+} = \frac{\cos e}{0^-}\right\} = +\infty$

4. $\lim\limits_{x\to e} \left(x + 2^{\frac{1}{1-\ln x}}\right) = \left\{e + 2^{\frac{1}{0}}\right\}$ granica nie istnieje

$\lim\limits_{x\to e^-} \left(x + 2^{\frac{1}{1-\ln x}}\right) = \left\{e + 2^{\frac{1}{0^+}} = e + 2^{+\infty} = e + +\infty\right\} = \infty$

$\lim\limits_{x\to e^+} \left(x + 2^{\frac{1}{1-\ln x}}\right) = \left\{e + 2^{\frac{1}{0^-}} = e + 2^{-\infty} = e + 0\right\} = e$

$\frac{1}{2^\infty} = \frac{1}{\infty} = 0$